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3 (Sem–5/CBCS) MAT HE 4/5/6

2021

(Held in 2022)

MATHEMATICS

(Honours Elective)

Answer the Questions from any one Option.

OPTION–A

Paper : MAT–HE–5046

(Linear Programming)

DSE(H)–2

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : $1 \times 10 = 10$

   (a) A basic feasible solution whose variables are.

      (i) degenerate

      (ii) nondegenerate

Contd.

(iii) non-negative

(iv) None of the above

(Choose the correct answer)

(b) The inequality constraints of an LPP can be converted into equation by introducing

(i) negative variables

(ii) non-degenerate B.F.

(iii) slack and surplus variables

(iv) None of the above

(Choose the correct answer)

(c) A solution of an LPP, which optimize the objective function is called

(i) basic solution

(ii) basic feasible solution

(iii) optimal solution

(iv) None of the above

(Choose the correct answer)

(d) What is artificial variable of an LPP ?

(e) Write the equation of line segment in $\mathbb{R}^n$.

(f) Define dual of a given LPP.

(g) What is pure strategy of game theory ?

(h) Is region of feasible solution to an LPP constitute a convex set ?

(i) Is every convex set in $\mathbb{R}^n$ a convex polyhedron also ?

(j) Is every boundary point an extreme point of a convex set ?

2. Answer the following questions : 2×5=10

(a) Show that the feasible solution

$x_1 = 1, x_2 = 0, x_3 = 1, z = 6$ to the system

$$\min \; Z = 2x_1 + 3x_2 + 4x_3$$

$$\text{s.t.} \quad x_1 + x_2 + x_3 = 2$$

$$x_1 - x_2 + x_3 = 2 \;, \quad x_i \geq 0$$

is not basic.

(b) A hyperplane is given by the equation

$$3x_1 + 2x_2 + 4x_3 + 7x_4 = 8$$

Find in which half space do the point $(-6, 1, 7, 2)$ lie.

(c) Find extreme points if any of the set $S = \{(x, y) : |x| \leq 1, \; |y| \leq 1\}$

(d) Show by an example that the union of two convex sets is not necessarily a convex set.

(e) If $x_1 = 2$, $x_2 = 3$, $x_3 = 1$ a BFS of the LPP

$$\max\ Z = x_1 + 2x_2 + 4x_3$$

$$\text{s.t.}\quad 2x_1 + x_2 + 4x_3 = 11$$

$$3x_1 + x_2 + 5x_3 = 14$$

$$x_1, x_2, x_3 \geq 0$$? Explain.

3. Answer *any four* questions : 5×4=20

(a) Prove that the set of all feasible solutions of an LPP is a convex set.

(b) Sketch the convex polygon spanned by the following points in a two-dimensional Euclidean space. Which of these points are vertices ? Express the other as the convex linear combination of the vertices

$$(0,0),\ (0,1),\ (1,0),\ \left(\frac{1}{2}, \frac{1}{4}\right).$$

(c) If $x_0 \in S$ where S is the set of all FS of the LPP min $Z = cx$, such that $Ax = b$, $x \geq 0$ minimize the objective function $Z = cx$, then show that x_0 also maximize the objective function $Z^* = (-c)x$ over S.

(d) Find the dual of the following LPP :

$$\min Z_p = x_1 + x_2 + x_3$$

s.t.
$$x_1 - 3x_2 + 4x_3 = 5$$
$$2x_1 - 3x_2 \leq 3$$
$$2x_2 - x_3 \geq 5$$
$$x_1, x_2, x_3 \geq 0$$

(e) Prove that the dual of a dual is a primal problem itself.

(f) Write the characteristics of an LPP in canonical form.

4. Answer *(a)* **or** *(b)*, *(c)* **or** *(d)*, *(e)* **or** *(f)*, *(g)* **or** *(h)* : 10×4=40

(a) Old hens can be bought for Rs. 2 each but young ones cost Rs. 5 each. The old hens lay 3 eggs per week and the young ones 5 eggs per week, each being worth 30 paise. A hen costs Re. 1 per week to feed. If I have only Rs. 80 to spend for hens, how many of each kind shall I buy to give a profit of more than Rs. 6 per week, assuming that I can not house more than 20 hens ? Formulate the LPP and solve by graphical method.

(b) Find all basic and then all the basic feasible solutions for the equations

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$
$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$

and determine the associated general convex combination of the extreme point solutions.

(c) State and prove the fundamental theorem of LPP.

(d) Solve by simplex method :

$$\begin{aligned} \max \quad & Z = 3x_1 + 5x_2 + 4x_3 \\ \text{s.t.} \quad & 2x_1 + 3x_2 \le 8 \\ & 3x_1 + 2x_2 + 4x_3 \le 15 \\ & 2x_2 + 5x_3 \le 10 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

(e) If in an assignment problem, a constant is added or substracted to every element of a row (or column) of the cost matrix $[c_{ij}]$, then prove that an assignment which minimizes the total cost for one matrix, also minimizes the total cost for the other matrix.

(f) Solve the following transportation problem :

To

		S_1	S_2	S_3	S_4	Supply
From	O_1	1	2	1	4	30
	O_2	3	3	2	1	50
	O_3	4	2	5	9	20
Demand		20	40	30	10	100

(g) For any zero-sum two-persons game where the optimal strategies are not pure and for which A's pay-off matrix is

$$\begin{array}{ccc} & & B \\ & & \begin{array}{cc} \mathrm{I}y_1 & \mathrm{II}y_2 \end{array} \\ A & \begin{array}{c} x_1\mathrm{I} \\ x_2\mathrm{II} \end{array} & \boxed{\begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array}} \end{array}$$

the optimal strategies are (x_1, x_2) and (y_1, y_2) then prove that

$$\frac{x_1}{x_2} = \frac{a_{22} - a_{21}}{a_{11} - a_{12}} \text{ and } \frac{y_1}{y_2} = \frac{a_{22} - a_{12}}{a_{11} - a_{21}} \text{ and}$$

the value of the game to A is given by

$$v = \frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

(h) Solve the game whose pay-off matrix is

$$\begin{bmatrix} -1 & -2 & 8 \\ 7 & 5 & -1 \\ 6 & 0 & 12 \end{bmatrix}$$

OPTION–B

Paper : MAT–HE–5056

(*Spherical Trigonometry and Astronomy*)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×10=10

   (a) State *one* fundamental difference between a spherical triangle and a plane triangle.

   (b) Define primary circle.

   (c) Define polar triangle and its primitive triangle.

   (d) State the third law of Kepler.

   (e) Explain what is meant by rising and setting of stars.

   (f) Write *any two* coordinate systems to locate the position of a heavenly body on the celestial sphere.

   (g) Define synodic period of a planet.

   (h) Mention *one* property of pole of a great circle.

(i) Just mention how a spherical triangle is formed.

(j) What is the declination of the pole of the ecliptic ?

2. Answer the following questions : $2\times5=10$

(a) Prove that section of a sphere by a plane is a circle.

(b) Discuss the effect of refraction on sunrise.

(c) Drawing a neat diagram, discuss how horizontal coordinates of a heavenly body are measured.

(d) Prove that the altitude of the celestial pole at any place is equal to the latitude of that place.

(e) Show that right ascension α and declination δ of the sun is always connected by the equation $\tan\delta = \tan\varepsilon \sin\alpha$, ε being obliquity of the ecliptic.

3. Answer ***any four*** of the following : $5\times4=20$

(a) Deduce Kepler's laws from Newton's law of gravitation.

(b) Show that the velocity of a planet in its elliptic orbit is $v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$ where $\mu = G(M+m)$ and a is the semi-major axis of the orbit.

(c) If z_1 and z_2 are the zenith distances of a star on the meridian and the prime vertical respectively, prove that

$$\cot\delta = \operatorname{cosec} z_1 \sec z_2 - \cos z_1$$

where δ is the star's declination.

(d) If H be the hour angle of a star of declination δ when its azimuth is A and H' when the azimuth is $(180° + A)$, show that

$$\tan\phi = \frac{\cos\frac{1}{2}(H'+H)}{\cos\frac{1}{2}(H'-H)}$$

(e) In an equilateral spherical triangle ABC, prove that $2\cos\frac{a}{2}\sin\frac{A}{2} = 1$.

(f) If ψ is the angle which a star makes at rising with the horizon, prove that $\cos\psi = \sin\phi\sec\delta$, where the symbols have their usual meanings.

4. Answer ***any four*** questions of the following : 10×4=40

(a) If the colatitude is C, prove that

$$C = x + \cos^{-1}(\cos x \sec y)$$

where $\tan x = \cot \delta \cos H$ and $\sin y = \cos \delta \sin H$,

H being the hour angle.

(b) In any spherical triangle ABC, prove that $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$. Also prove that $\frac{\sin(A+B)}{\sin C} = \frac{\cos a + \cos b}{1 + \cos c}$

(c) Define astronomical refraction and state the laws of refraction. Derive the formula for refraction as $R = k \tan \xi$, ξ being the apparent zenith distance of a heavenly body. Mention *one* limitation of this formula.

(d) On account of refraction, the circular disc of the sun appears to be an ellipse. Prove it.

(e) Derive Kepler's equation in the form $M = E - e \sin E$, where M and E are respectively mean anomaly and eccentric anomaly.

(f) Assuming the planetary orbits to be circular and coplanar, prove that the sidereal period P and the synodic period S of an inferior planet are related to the earth's periodic time E by

$$\frac{1}{S}=\frac{1}{P}-\frac{1}{E}$$

Calculate the sidereal period (in mean solar days) of a planet whose sidereal period is same as its synodic period.

(g) Prove that, if the fourth and higher powers of e are neglected,

$$E=M+\frac{e\sin M}{1-e\cos M}-\frac{1}{2}\left(\frac{e\sin M}{1-e\cos M}\right)^3$$

is a solution of Kepler's equation in the form.

(h) Derive the expressions to show the effect of refraction in right ascension and declination.

OPTION–C

Paper : MAT–HE–5066

(Programming in C)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

   (a) Write *any two* special characters that are used in *C*.

   (b) Mention *two* data types that are used in *C* language.

   (c) For $x=2$, $y=5$, write the output of the *C* function 'pow (x, y)'.

   (d) Convert the mathematical expression
   $$z = e^x + \log y + \sqrt{1+x}$$
   into *C* expression.

   (e) Write the utility of clrscr ( ) function.

   (f) Write a difference between local variable and global variable.

   (g) Write the *C* library function which can evaluate $|x|$.

2. Answer the following questions : 2×4=8

(a) Write the difference between 'assignment' and 'equality'.

(b) How does 'x + +' differ from '+ + x' ?

(c) What is a string constant ? Give an example.

(d) Write *four* relational operators that are used in *C*.

3. Answer ***any three*** parts : 5×3=15

(a) Explain artihmetic and logical operators in *C* with suitable examples.

(b) List three header files that are used in *C*. Also write their utilities. 3+2=5

$A = 5$; $B = 3$

$A = A + B$;

$B = A - B$;

$A = A - B$;

Write the output of *A* and *B* from the above program segment in *C*.

(c) Write a *C* program to find the sum of all odd integers between 1 and *n*.

(d) Write the general form of do-while loop and explain how it works with the help of a suitable example.

(e) Write the utility of 'break' and 'continue' statements with the help of suitable examples.

4. Why are arrays required in C programming ? How are one-dimensional arrays declared and inputs given to array ? Explain briefly with example. Write a program to read given n numbers and then find the sum of all positive and negative numbers. 2+3+5=10

Or

How are two-dimensional arrays declared ? Write a C program to read a 3×3 matrix and print the same as output. Hence write a C program to read a 3×3 matrix, print its transpose and write the determinants of both. 1+4+5=10

5. Write a C program for each of the following :

(a) To evaluate the function 5

$$f(x) = x^2 + 2x - 10, \ x \geq 0$$
$$= |x|, x < 0$$

(b) To find the biggest of three numbers. 5

Or

Explain with example the 'if' statement and nested 'if' statement in *C*. Write a *C* program to find the roots of a quadratic equation $ax^2 + bx + c = 0$, for all possible values of a, b, c. 5+5=10

6. What is the basic difference between 'Library functions' and 'User-defined functions' ? Mention *two* advantages of using 'User-defined functions'. How are such functions declared and called in a program ? Write a *C* program using function to find the biggest of three numbers. 1+2+2+5=10

Or

Write a *C* programme that reads a number, obtains a new number by reversing the digits of the given number, and then determine the gcd of the two numbers. To build the programme, use two functions — one to find gcd and another to reverse the digits. 10