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3 (Sem–4/CBCS) MAT HC 2

2022

MATHEMATICS

(Honours)

Paper : MAT–HC–4026

(Numerical Methods)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** questions : $1\times7=7$

   (a) What do you mean by an algorithm ?

   (b) What is the underlying theorem of bisection method ?

   (c) Write the iterative formula of secant method for solving an equation $f(x)=0$.

   (d) Consider the system of equations $Ax=b$. In which method, the matrix A can be decomposed into the product of two triangular matrices ?

Contd.

(e) Name one iterative method for solving a system of linear equations.

(f) Write the iterative formula of Newton-Raphson method to find the square root of 15.

(g) What do you mean by interpolating polynomial ?

(h) Show that $\Delta = E - 1$.

(i) What do you mean by numerical differentiation ?

(j) Write the formula for second order central difference approximation to the first derivative.

2. Answer ***any four*** questions : 2×4=8

(a) Examine whether the fixed point iteration method is applicable for finding the root of the equation :

$$2x = \sin x + 5.$$

(b) Define rate of convergence and order of convergence of a sequence.

(c) Prove that $\mu = \left(1 + \frac{\delta^2}{4}\right)^{1/2}$ where μ and δ are average and central difference operators.

(d) Verify that the following equation has a root on the interval $(0,1)$:

$$f(x) = ln(1+x) - \cos x = 0 .$$

(e) If $P_1(x) = a_0 + a_1 x$ such that $P_1(x_0) = f_0$ and $P_1(x_1) = f_1$, then obtain an expression for $P_1(x)$ in terms of x_i's and f_i's $(i = 0, 1)$.

(f) Show that $\delta = \nabla(1-\nabla)^{-\frac{1}{2}}$.

(g) What do you mean by degree of precision of a quadrature rule ? If a quadrature rule $I_n(f)$ integrates $1, x, x^2$ and x^3 exactly, but fails to integrate x^4 exactly, then what will be the degree of precision of $I_n(f)$?

(h) Mention briefly about the use of Euler's method.

3. Answer ***any three*** questions : 5×3=15

(a) Give a brief sketch of the method of false position.

(b) Give the geometrical interpretation of Newton-Raphson method.

(c) Construct an algorithm for the secant method.

(d) Show that an LU decomposition is unique up to scaling by a diagonal matrix.

(e) Discuss about the advantages and disadvantages of Lagrange's form of interpolating polynomial.

(f) Given $f(2)=4,\ f(2.5)=5.5$, find the linear interpolating polynomial using Lagrange's interpolation. Hence find an approximate value of $f(2.2)$.

(g) Derive the closed Newton-Cotes quadrature formula corresponding to $n=1$. Why is this formula called trapezoidal rule ?

(h) Evaluate $\int_0^1 tan^{-1} x\, dx$ using Simpson's $\frac{1}{3}$rd rule.

4. Answer ***any three*** questions : 10×3=30

(a) Perform five iterations of the bisection method to obtain the smallest positive root of the equation :

$$f(x)=x^3-5x+1=0.$$

(b) Apply Newton-Raphson method to determine a root of the equation :

$$f(x) = \cos x - xe^{x} = 0.$$

Taking the initial approximation as $x_0 = 1$, perform five iterations.

(c) Form an LU decomposition of the following matrix :

$$A = \begin{pmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 5 & 8 & -2 \end{pmatrix}$$

(d) Find the order of convergence of the iterative method $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$ to compute an approximation to the square root of a positive real number a. To find the real root of $x^3 - x - 1 = 0$ near $x = 1$, which of the following iteration functions give convergent sequences ?

(i) $x = x^3 - 1$

(ii) $x = \frac{x+1}{x^2}$

(e) Construct the difference table for the sequence of values :

$$f(x) = (0, 0, 0, \varepsilon, 0, 0, 0).$$

where ε is an error. Also show that —

(i) the error spreads and increases in magnitude as the order of differences is increased;

(ii) the errors in each column have binomial coefficients.

(f) Let $x_0 = -3$, $x_1 = 0$, $x_2 = e$ and $x_3 = \Pi$. Determine formulas for the Lagrange's polynomials $L_{3,0}(x)$, $L_{3,1}(x)$, $L_{3,2}(x)$ and $L_{3,3}(x)$ associated with the given interpolating points.

(g) For the function $f(x) = ln\,x$, approximate $f'(3)$ using —

(i) first order forward difference, and

(ii) first order backward difference approximation formulas.

[Starting with step size $h = 1$, reduce it by $\frac{1}{10}$ in each step until convergence.]

5+5=10

(h) Solve the initial value problem :

$$\frac{dx}{dt} = 1 + \frac{x}{t}, \; 1 \le t \le 2.5$$

$$x(1) = 1,$$

using Euler's method with step size $h = 0.5$ and find an approximate value of $x(2.5)$.
