Total number of printed pages–8

3 (Sem–4/CBCS) PHY HC 1

2022

PHYSICS

(Honours)

Paper : PHY–HC–4016

(Mathematical Physics–III)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** questions of the following : $1\times7=7$

   (a) What is the argument of $-3i$?

   (b) Express $f(z)=z^2$ in the form of $u(x, y)+iv(x, y)$.

   (c) What is singular point of an analytic function ?

Contd.

(d) Evaluate $\delta_q^p A_s^{qr}$.

(e) State the shifting property of Fourier transform (FT).

(f) Find the residue of the complex function $f(z) = \frac{1}{z^2 + 1}$ at the pole $z = i$.

(g) Show that $L(1) = \frac{1}{s}, s > 0$.

(h) What is rank of a tensor? Give *one* example of a zero rank tensor.

(i) Define Fourier inverse transform.

(j) Write the polar form of a complex number.

2. Answer ***any four*** of the following questions : 2×4=8

(a) Check whether the function *log z* is analytic or not.

(b) Plot the complex number $e^{(1-\pi/6i)}$ in Argand diagram.

(c) Prove that the contraction of the tensor A_m^l is invariant.

(d) Obtain the Fourier transform of the function

$$f(x)=\begin{cases} x, & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$$

(e) Using the property of Levi-Civita symbol prove that $\vec{A}\times\vec{B}=-\left(\vec{B}\times\vec{A}\right)$.

(f) If $L[f(x)]=\bar{f}(s)$, then show that $L\left[e^{ax}f(x)\right]=\bar{f}(s-a)$.

(g) Evaluate the integral $\oint\frac{dz}{z}$ around a unit circle.

(h) Expand the function

$f(z)=\frac{1}{z+1}$, about $z=1$ in Taylor series up to two terms.

3. Answer ***any three*** questions of the following : 5×3=15

(i) Find the value of the integral $\int_0^{1+i}\left(x-y-ix^2\right)dz$, along real axis from $z=0$ to $z=1$ and then along the line parallel to imaginary axis from $z=1$ to $z=1+i$.

(ii) State and prove Cauchy's integral formula.

(iii) Obtain the Fourier sine and cosine transform of the function

$$f(x)=\begin{cases}1, & 0<x<\pi/2\\ 0, & x>\pi/2\end{cases}$$

(iv) What is Kronecker delta? Show that it is a mixed tensor of rank 2. 2+3=5

(v) Find the Laplace transform of the function $f(t)=sinat$.

(vi) Show that $|z_1\cdot z_2|=|z_1|\cdot|z_2|$ and $Arg(z_1\cdot z_2)=Arg(z_1)+Arg(z_2)$.

(vii) What are raising and lowering of indices of a tensor? Prove that the raising and lowering operation of indices are reciprocal to each other. 2+3=5

(viii) Evaluate $\oint_C \frac{\cos z}{z} dz$, where C is an ellipse given by $9x^2 + 4y^2 = 1$, using Cauchy's integral formula. 5

4. Answer ***any three*** of the following questions : 10×3=30

(a) (i) Show that if $f(z) = u + iv$ is an analytic function and $\vec{F} = \hat{i}v + \hat{j}u$ is a vector, then $div\,\vec{F} = 0$ and $curl\,\vec{F} = 0$ are equivalent to Cauchy-Reimann equations. 6

(ii) State and prove quotient law of tensors. 4

(b) (i) The Laplace transform of $\sin 3t$ is $\frac{3}{S^2+9}$ and the Laplace transform of $\cos 5t$ is $\frac{S}{S^2+25}$. Find the Laplace transform of $5\sin 3t + 3\cos 5t$ using linearity property of Laplace transform. 5

(ii) Find the inverse Laplace transform of $\frac{4S+5}{(S-1)^2(S+2)}$. 5

(c) (i) If A_λ is a covariant tensor of rank 1, show that $\frac{\partial A_\lambda}{\partial x_\mu}$ is not a tensor. 3

(ii) Prove the following identities : 2+2+3=7

(a) $\delta_{ii} = 3$

(b) $\delta_{ik}\varepsilon_{ikm} = 0$

(c) $\varepsilon_{iks}\varepsilon_{mps} = \delta_{im}\delta_{kp} - \delta_{ip}\delta_{km} = 0$

(d) State and prove Fourier integral theorem.

(e) (i) Using the method of residues, show that $\int_0^\infty \frac{dx}{x^4+1} = \frac{\pi\sqrt{2}}{4}$. 6

(ii) Express the complex number $1+2i/1-3i$ in $r(cos\theta + i\,sin\theta)$ form. 4

(f) Evaluate ***any two*** of the following integrals by contour integration :

5×2=10

(i) $\int_0^{\infty} \frac{dx}{x^2+1}$

(ii) $\int_{-\infty}^{\infty} \frac{\sin x}{x} dx$

(iii) $\int_{-\infty}^{+\infty} \frac{e^{ax}}{1+e^x} dx$

(g) Solve the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ under the conditions that, $y(x, 0) = 0$, $y'(x, 0) = 0, x > 0$ and $y(0, t) = t$, $\lim_{x \to \infty} y(x, t) = 0, t \geq 0$.

(h) (i) What is residue of a complex function? State and prove Cauchy's residue theorem.

1+1+4=6

(ii) Show that any contravariant or covariant tensor of the second order can be resolved into symmetric and antisymmetric parts. 4
