2018

MATHEMATICS

(Major)

Paper : 4.2

(Mechanics)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : $1\times10=10$

(a) A system of forces in one plane acts on a rigid body and the moments of the system about three non-collinear points in the plane are α, β and γ. Write the condition satisfied by α, β, γ under which the system is equivalent to a single force.

(b) When two parallel forces cannot be compounded into a single resultant force?

(c) Write down the relation between the angle of friction and the coefficient of friction.

(d) State the energy test for stability.

(e) What is the geometrical representation of simple harmonic motion?

(f) State Hooke's law of elasticity.

(g) State two forces that can be omitted in forming equation of virtual work.

(h) Define wrench. What is intensity of wrench?

(i) Define areal velocity of a particle moving in a curve.

(j) Which law of Kepler is related to areal velocity of a planet? State the law.

2. Answer the following questions : $2 \times 5 = 10$

(a) Two men have to carry a block of stone of weight 70 kg on a light plank. Where must the block be placed so that one of the men should bear the weight of 10 kg more than the other?

(b) Equal forces act along the coordinate axes and along the line

$$\frac{X-\alpha}{l}=\frac{Y-\beta}{m}=\frac{Z-\gamma}{n}$$

Find the elements $(X, Y, Z; L, M, N)$ of the system.

(c) A point describes a cycloid $S = 4a\sin\psi$ with uniform speed v. Find its acceleration at any point s.

(d) If the angular velocity of a point moving in a plane curve be constant about a fixed origin, show that its transverse acceleration varies as its radial velocity.

(e) Show that the conservation of energy for a particle of mass m moving in a central force field can be expressed as

$$\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)-\int f(r)\,dr=E$$

where E is constant.

3. Answer any *four* of the following questions : 5×4=20

(a) If three forces acting upon a rigid body be represented in magnitude, direction sense and line of action by the sides of a triangle taken in order, then show that the forces are equivalent to a couple whose moment is equal to twice the area of the triangle.

(b) State and prove the principle of virtual work for a system of coplanar forces acting at different points of a rigid body.

(c) Three forces X, Y, Z act along the three straight lines $y = b$, $z = -c$; $z = c$, $x = -a$ and $x = a$, $y = -b$ respectively. Show that they will have a single resultant if

$$\frac{a}{X} + \frac{b}{Y} + \frac{c}{Z} = 0$$

Also show that the equations of its line of action are any two of the following three equations :

$$\frac{y}{Y} - \frac{z}{Z} - \frac{a}{X} = 0, \ \frac{z}{Z} - \frac{x}{X} - \frac{b}{Y} = 0, \ \frac{x}{X} - \frac{y}{Y} - \frac{c}{Z} = 0$$

(d) Two bodies of masses M and M' respectively are attached to the lower end of an elastic string whose upper end is fixed and are hung at rest; M' falls off. Show that the distance of M from the upper end of the string at time t is

$$a + b + c\cos\left(\sqrt{\frac{g}{b}}\,t\right)$$

where a is the unstretched length of the string, b and c the distances by which it would be stretched when supporting M and M' respectively.

(e) A particle of mass m is acted upon by a force $m\mu\left(x+\frac{a^4}{x^3}\right)$ towards the origin O. If it starts from rest at a distance a from O, show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$.

(f) Under the influence of a central force P at a point O, a particle describes a circle which passes through the pole O. Find the law of force.

4. (a) Prove that the centre of gravity of a body is unique. 2

(b) Find the centre of gravity of the area bounded by the parabola $y^2 = 4ax$, the axis of x and the latus rectum. 4

(c) A uniform cubical box of edge a is placed on the top of a fixed sphere, the centre of the face of the cube being in contact with the highest point of the sphere. What is the least radius of the sphere for which equilibrium will be stable? 4

5. (a) Establish the following statement : 5

Any system of coplanar forces acting on a rigid body can ultimately be reduced either to a single force or to a single couple unless it is in equilibrium.

(*b*) A heavy uniform rod of length $2a$ rests in equilibrium against a smooth vertical wall and being placed upon a peg at a distance h from the wall. Show that the inclination of the rod to the horizon θ is given by

$$\cos^3\theta = \frac{h}{a}$$ 5

Or

A solid hemisphere of weight W rests in limiting equilibrium with its curved surface on a rough inclined plane, and the plane face is kept horizontal by a weight P attached to a point in the rim. Prove that the coefficient of friction is

$$\frac{P}{\sqrt{W(2P+W)}}$$ 5

6. Answer *either* (*a*) *or* (*b*) :

(*a*) (*i*) Find the centre of gravity of the arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ which is in the positive quadrant.

(*ii*) A body consisting of a cone and a hemisphere on the same base rests on a rough horizontal table, the hemisphere being in contact with the table. Show that the greatest height of the cone, so that the equilibrium may be stable is $\sqrt{3}$ times the radius of the hemisphere.

5+5=10

(b) (i) Find the centre of gravity of a solid formed by the revolution of a quadrant of an ellipse about the minor axis.

(ii) A heavy uniform rod rests with one end against a smooth vertical wall and with a point in its length resting on a smooth peg; find the position of equilibrium and show that it is unstable. 5+5=10

7. (a) A particle of mass m is projected vertically upwards under gravity, the resistance of air being mk times the velocity. Show that the greatest height attained by the particle is

$$\frac{V^2}{g}[\lambda - \log(1+\lambda)]$$

where V is the terminal velocity of the particle and λV is the initial velocity. 5

(b) A particle starts from rest and slides from the highest point of a smooth vertical circle and falls under gravity. Determine where the particle will leave the circle. 5

Or

Establish the formula

$$P = m\frac{dv}{dt} + \lambda u$$

for the motion of a particle of varying mass $m(t)$ with velocity v under a force P, matter being emitted at a constant rate λ with velocity u relative to the particle. 5

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