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3 (Sem–3/CBCS) PHY HC 1

2023

PHYSICS

(Honours Core)

Paper : PHY–HC–3016

(Mathematical Physics–II)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1\times7=7$

   (a) Define the ordinary point of a second order differential equation.

   (b) Show that $P_0(x)=1$.

   (c) Write the Laplace equation in spherical polar co-ordinate system.

   (d) A partial differential equation has

      (i) one independent variable

Contd.

(ii) more than one dependent variable

(iii) two or more independent variables

(iv) no independent variable.

(Choose the correct option)

(e) If A and B are two square matrices of order n, show that

$$Trace\,(A+B) = Trace\,A + Trace\,B$$

(f) Which one of the following is the value of $\Gamma\left(\frac{1}{2}\right)$?

(i) $\sqrt{\pi/2}$

(ii) $\sqrt{\pi}$

(iii) π

(iv) $\sqrt{\pi}/2$

(g) Define self adjoint of a matrix.

2. Answer the following questions : 2×4=8

(a) Check the behaviour of point $x=0$ for the differential equation

$$\frac{d^2y}{dx^2} - \frac{6}{x}y = 0$$

(b) If $\int_{-1}^{+1} P_n(x)\,dx = 2$, find the value of n.

(c) Express the Fourier series in complex form.

(d) Verify the matrix

$$A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$ is a unitary matrix.

3. Answer **any three** of the following questions : 5×3=15

(a) Find the power series solution of the following differential equation :

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

about $x = 0$.

(b) Define Gamma function. Show that

$$\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$$ 1+4=5

(c) Establish the following recurrence formula for Legendre polynomial $P_n(x)$

$$nP_n(x) = (2n-1)xP_{n-1}(x) - (n-1)P_{n-2}(x)$$

(d) Show that the Fourier expansion of the function $f(x) = x,\ 0 < x < 2\pi$ is

$$x = \pi - 2\left[\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \ldots\right]$$

(e) What is eigenvalue of a matrix? Show that the eigenvalues of Hermitian matrix are real. 1+4=5

4. Answer ***any three*** of the following questions : 10×3=30

(a) (i) If $P_n(x)$ be the polynomial of Legendre's differential equation, show that

$$P_n(x) = \frac{1}{2^n.n!} \cdot \frac{d^n}{dx^n}\left(x^2-1\right)^n \qquad 6$$

(ii) Prove that 4

$$\int_{-1}^{+1} xP_n(x)P_{n-1}(x)\,dx = \frac{2n}{4n^2-1}$$

(b) (i) What is Beta function ? Show that

(a) $\beta(1,1) = 1$

(b) $\beta(m,n) = \beta(n,m)$ 1+1+3=5

(ii) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, show that

$A^2 - 4A - 5I = 0$ where I and 0 are the unit matrix and the null matrix of order 3 respectively. Also use this result to find A^{-1}. 3+2=5

(c) Find the Fourier series expansion of

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Also show that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$ 6+4=10

(d) (i) Diagonalize the following matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$ 5

(ii) Show that the generating function for Hermite polynomial $H_n(x)$, for integral n and real value of n is given by

$$e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x)$$ 5

(e) (i) Write the one dimensional diffusion equation (heat flow equation) and find the general solution of the same by the method of separation of variable.

1+7=8

(ii) For the Pauli spin matrices

$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ and

$\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ show that

$$[\sigma_1, \sigma_2] = 2i\sigma_3$$ 2

(f) (i) Write the Orthogonality conditions of *sine* and *cosine* functions.

1½+1½=3

(ii) A square wave function is represented as

$$f(x)=\begin{cases}0, & \text{for } -\pi < x < 0\\ h, & \text{for } 0 \le x < \pi\end{cases}$$

Draw the graphical representation of the wave function and expand the same in Fourier series.

1+6=7