14 (ECO– 1) 1036

2023

ECONOMICS

Paper : ECO– 1036

(Quantitative Orientation)

Full Marks : 80

Time : Three hours

The figures in the margin indicate
full marks for the questions.

FIRST HALF

(Marks : 50)

1.   Answer the following :   4×3=12

(a)   What is the economic interpretation of
the Lagrange multiplier in the
optimization problem of $u = u(xy)$
subject to the budget constraint
$B = XP_x + YP_y$.

(b)   What does it mean for a matrix to be of
full rank ? Are there any condition under
which the rank of a matrix might be zero,
and what does that imply ?

(c) What is the significance of the 'row sum' of an input-output table ? What do columns represent in an input-output table ?

2. Answer *any three* of the following : 8×3=24

   (a) What is the difference between the Hessian determinant and the bordered Hessian determinant ? Discuss the significance of the second-order conditions in equality-constrained optimization problems. How does the bordered Hessian determinant help in analyzing these conditions ?   2+4+2=8

   (b) Discuss the concept of critical point in optimization. How are critical points related to maxima and minima ? What is the significance of the sufficient conditions for identifying the nature of local optimization ?   3+2+3=8

   (c) Illustrate the economic interpretation of a definite integral by using it for calculating consumer surplus. Suppose the supply function for selling x units is given by the equation $S(x) = 250 + 5x$; the demand function for x units is given by $D(x) = 100 - 10x$. Find the producer's and consumer's surplus.   3+5=8

(d) (i) If AR = 15 and MR = 5, calculate the price elasticity of demand.

(ii) The cost function of a monopolist is $C = 40 + 12X$ and its demand function is $P = 60 - 3X$. Calculate the profit-maximizing output.

(iii) Integrate : $\int \dfrac{x^2}{x+2}\, dx$

2+3+3=8

(e) Draw functions $f(x)$ when –
(i) $f'(x) > 0$ and $f''(x) > 0$
(ii) $f'(x) > 0$ and $f''(x) < 0$
(iii) $f'(x) < 0$ and $f''(x) < 0$
(iv) $f'(x) < 0$ and $f''(x) > 0$

2+2+2+2=8

3. Answer **any one** of the following : 14×1=14

(a) (i) For the following input-output table, calculate the technology matrix and also write the balance equation for the two sectors :

Sector	Industry	Service	Final Demand
Industry	50	150	200
Service	100	75	100

Also, determine the total output if the final demand changes to 180 for the industry and 100 for the service sector. 10

(ii) Suppose a profit-maximizing monopolist is producing 800 units of output and is charging a price of Rs.40 per unit. If the elasticity of demand for the products is –2, find out the marginal cost of the last unit produced.　　　　　4

(b) *(i)* Assume the rate of investment is described by the function $I_t = 3t^{1/3}$ and the stock of capital at $t = 0$ is $K(0) = 25$. Find the time path of capital stock K, and also find the amount of capital accumulation during the time intervals $[0, 5]$ and $[4, 5]$ respectively.　　　　　5

(ii) Derive the equilibrium conditions for a rational consumer to achieve its equilibrium by maximizing $U = f(x_1, x_2)$ subject to

$B = x_1 p_1 + x_2 p_2$,

where p_1 and p_2 are the prices of x_1 and x_2.　　　　　9

SECOND HALF

(Marks : 30)

4. Answer the following questions :   4×2=8

    (a) *(i)* A die is thrown at random. What is the mathematical expectation on it ?   2

        (ii) State Bayes' theorem.   2

    (b) *(i)* What is the shape of the theoretical distribution of the mean if the parent population is normal ? What is the shape of the distribution if the parent population is not normal but the sample size, $n \geq 30$?

                                        1+1=2

        (ii) Show that the total number of conditions for the mutual independence of n events

           $A_1, A_2, ..., A_n$ is $2^n - 1 - n$.   2

5. Answer ***any two*** of the following questions :   11×2=22

    (a) *(i)* If A, B and C are mutually exclusive and exhaustine events, and

$$P(A) = \frac{1}{2}P(B) \text{ and } P(B) = \frac{2}{3}P(C),$$

          find $P(A), P(B)$ and $P(C)$.   3

(ii) In a bag, there are 5 white and 3 black balls. What is the probability that if they are drawn out one after another without replacement, the first ball drawn will be white, the second black, the third white and again the fourth a black one ?   4

(iii) The first three moments of a distribution about the value 67 of the variable are 0.45, 8.73 and 8.91. Calculate the second and third central moments, and moment co-efficient of skewness. Indicate the nature of the distribution and sketch the distribution.   1+1+1+1=4

(b) *(i)* State the working rule to find mode of the Binomial distribution.   2

(ii) Under what conditions the Poisson distribution be used as a limiting case of Binomial distribution ? Deduce Poisson distribution as an approximation of Binomial distribution.   2+4=6

(iii) If X and Y are independent Poisson variates such that

$P(X = 1) = P(X = 2)$ and
$P(Y = 2) = P(Y = 3)$, find the variance of $(X - 2Y)$.   3

(c) *(i)* If a discrete random variable X follows a Binomial distribution with parameters n and p, then obtain Moment Generating Function of $X, M_X(t)$ where t is any real number. 4

(ii) The hourly wages of 1,000 workers are normally distributed around a mean of Rs.70 and with a standard deviation of Rs.5. Estimate the number of workers whose hourly wages will be

1. between Rs.69 and Rs.72

2. more than Rs.75

3. Also estimate the lowest hourly wages of the 100 highest paid workers.

2+2+3=7

[Given, $P(0 < z < 0.2) = 0.0793$

$P(0 < z < 0.4) = 0.1554$

$P(0 < z < 1.28) = 0.40$ (approx)]