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3 (Sem–6/CBCS) MAT HC 2

2024

MATHEMATICS

(Honours Core)

Paper : MAT–HC–6026

(Partial Differential Equations)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following: 1×7=7

   (i) Under which of the following conditions does arbitrary constant elimination usually produce more than one partial differential equation of order one ?

   (a) The number of arbitrary constants is less than that of independent variables

(b) The number of arbitrary constants equals the number of independent variables

(c) The number of arbitrary constants is more than that of independent variables

(d) Both (a) and (b)

(Choose the correct answer)

(ii) State True or False :

$\left(x^2 - yz\right)p + \left(y^2 - zx\right)q = z^2 - xy$ is a first order quasi-linear partial differential equation.

(iii) The order of $p\tan y + q\tan x = \sec^2 z$ is ______.

(iv) The Charpit's method is used for

(a) general solution

(b) complete solution

(c) singular solution

(d) complete integral

(Choose the correct answer)

(v) Jacobi's auxiliary equations for

$p_1x_1 + p_2x_2 - p_3^2 = 0$ are ______.

(vi) What are the characteristic equations of $u_x - u_y = u$?

(vii) The equation $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 0$ is

(a) parabolic for $x \neq 0$ and $y \neq 0$ only

(b) parabolic for $x = 0$ and $y = 0$ only

(c) parabolic everywhere

(d) parabolic nowhere

(Choose the correct answer)

2. Answer in short : 2×4=8

(i) Consider an equation of the form

$$a(x,y,u)u_x + b(x,y,u)u_y = c(x,y,u),$$

where its coefficients a, b and c are functions of x, y and u. Is it linear ? Justify your answer.

(ii) Eliminate the arbitrary function f from $z = x^n f\left(\frac{y}{x}\right)$ to form a partial differential equation.

(iii) Mention when Jacobi's method is used. Name an advantage of Jacobi's method over Charpit's method.

(iv) Construct an example of a partial differential equation that is elliptic in one domain but hyperbolic in another.

3. Answer ***any three*** : 5×3=15

(i) Find the partial differential equation that all surfaces of revolution satisfy with the z-axis as the axis of symmetry, along with a suitable explanation.

(ii) Find the general solution of the differential equation

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y)z.$$

(iii) Find the integral surface of the equation

$$(x-y)y^2p+(y-x)x^2q=(x^2+y^2)z$$

through the curve $xz = a^3$, $y = 0$.

(iv) Reduce the equation

$$u_x + 2xyu_y = x$$

to canonical form, and obtain the general solution.

(v) Discuss the general solution of $Au_{xx}+Bu_{xy}+Cu_{yy}=0$ with constant coefficients in hyperbolic case.

4. Answer the following : 10×3=30

(i) Discuss briefly the essential steps in Charpit's method for solving partial differential equations. Use this method to solve the equation $p = (z + qy)^2$.

Or

Show that the only integral surface of the equation $2q(z-px-qy)=1+q^2$ which is circumscribed about the paraboloid $2x = y^2 + z^2$ is the enveloping cylinder which touches it along its section by the plane $y + 1 = 0$.

(ii) Describe in brief the key components of the 'method of separation of variables'. Use this method suitably to solve the equation $u_x + u = u_y$, $u(x, 0) = 4e^{-3x}$.

Or

Use $v = \ln u$ and $v(x, y) = f(x) + g(y)$ to solve the equation $x^2u_x^2 + y^2u_y^2 = u^2$.

Also, discuss briefly the approach adopted to solve the above equation.

(iii) Consider the wave equation

$u_{tt} - c^2u_{xx} = 0$, c is constant.

Establish that any general solution of this equation can be expressed as the sum of two waves, one travelling to the right with constant velocity c and the other travelling to the left with the same velocity c.

Or

Find the general solution of the following equations :

(a) $yu_{xx} + 3yu_{xy} + 3u_x = 0,\ y \neq 0$

(b) $4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$
