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3 (Sem-5/CBCS) MAT HE 4/5/6

2024

MATHEMATICS

(Honours Elective)

Answer the Questions from any one Option.

OPTION - A

Paper : MAT-HE-5046

(Linear Programming)

Full Marks : 80

Time : Three hours

OPTION - B

Paper : MAT-HE-5056

(Spherical Trigonometry and Astronomy)

Full Marks : 80

Time : Three hours

OPTION - C

Paper : MAT-HE-5066

(Programming in C)

Full Marks : 60

Time : Three hours

**The figures in the margin indicate
full marks for the questions.**

Contd.

OPTION - A

Paper : MAT-HE-5046

(Linear Programming)

Full Marks : 80

1. Choose the correct answer : $1 \times 10 = 10$

(i) The optimal value of the objective function of the Linear Programming Problem (LPP),

$$\text{Minimize } Z = 3x_1 + 2x_2$$

$$\text{subject to } x_1 - x_2 \leq 1$$

$$x_1 + x_2 \geq 3$$

$$x_1, x_2 \geq 0$$

is -

(a) 5

(b) 6

(c) 7

(d) 8

- (ii) If the objective function of a LPP assumes its optimal value at more than one extreme points of the convex set of its feasible solutions, then
- (a) the LPP has no solution
 - (b) there exists at least one basic feasible solution which is not an extreme point
 - (c) the number of extreme points of the feasible region must exceed the number of basic feasible solutions
 - (d) every convex combination of these extreme points gives the optimal value of the objective function
- (iii) A basic solution to a system of linear equations is called degenerate, if -
- (a) none of the basic variables vanish
 - (b) exactly one of the basic variables vanish
 - (c) one or more of the basic variables vanish
 - (d) it is also a feasible solution

- (iv) Which of the following is not correct ?
- (a) The graphical approach to a LPP is most suitable when there are only two decision variables.
 - (b) Decision variables in a LPP may be more or less than the number of constraints.
 - (c) All the constraints and decision variables in a LPP must be of either " \leq " or " \geq " type.
 - (d) All decision variables in a LPP must be non-negative
- (v) Which of the following is not associated with a LPP ?
- (a) Proportionality
 - (b) Uncertainty
 - (c) Additivity
 - (d) Divisibility
- (vi) A necessary and sufficient condition for a basic feasible solution to a minimization LPP to be optimal is that for all j -
- (a) $z_j - c_j \leq 0$
 - (b) $z_j - c_j \geq 0$
 - (c) $z_j - c_j > 0$
 - (d) $z_j - c_j < 0$

(vii) Choose the incorrect statement :

- (a) If the primal is a maximization problem, its dual will be a minimization problem.
- (b) The primal and its dual do not have the same number of variables.
- (c) Corresponding to every unrestricted primal variable there is an equality dual constraint.
- (d) For an unbounded primal problem, its dual has a feasible solution.

(viii) The total transportation cost to the initial feasible solution to the transportation problem

	D_1	D_2	D_3	D_4	
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
	20	40	30	10	

obtained by least cost method is -

- (a) 100
- (b) 180
- (c) 310
- (d) 796

- (ix) The assignment problem is a special case of transportation problem in which the number of origins –
- (a) equals the number of destinations
 - (b) is greater than the number of destinations
 - (c) is less than the number of destinations
 - (d) is less than or equal to the number of destinations
- (x) In a two-person zero-sum game,
- (a) if the optimal solution requires one player to use a pure strategy, then the other player must also do the same
 - (b) gain of one player is exactly matched by a loss to the other player so that their sum is equal to zero
 - (c) the game is said to be fair if both the players have equal number of strategies
 - (d) the player having more strategies to play is said to dominate the other player

2. Answer the following questions : $2 \times 5 = 10$

(a) Examine the convexity of the set

$$S = \{ (x_1, x_2, x_3) : 2x_1 - x_2 + x_3 \leq 4 \}$$

(b) Explain the use of artificial variables in Linear Programming. Name *two* methods generally employed for the solution of LPP having artificial variables.

(c) Write the dual of the following LPP :

$$\text{Maximize } Z = 4x_1 + 7x_2$$

$$\text{subject to } 3x_1 + 5x_2 \leq 6$$

$$x_1 + 2x_2 \leq 8$$

$$x_1, x_2 \geq 0$$

(d) Give the mathematical formulation of a transportation problem.

(e) Find the saddle point of the pay-off matrix –

	B		
A	2	4	5
	10	7	8
	4	5	6

3. Answer **any four** of the following : $5 \times 4 = 20$

(a) Solve the following LPP graphically :

$$\text{Minimize } Z = 3x_1 + 5x_2$$

$$\text{subject to } 2x_1 + 3x_2 \geq 12$$

$$-x_1 + x_2 \leq 3$$

$$x_1 \leq 4$$

$$x_2 \geq 3$$

$$x_1, x_2 \geq 0$$

(b) Reduce the feasible solution $x_1 = 2$,
 $x_2 = 4$, $x_3 = 1$ to the system of equations

$$2x_1 - x_2 + 2x_3 = 2$$

$$x_1 + 4x_2 = 18$$

to a basic feasible solution.

(c) Use simplex method to solve the LPP :

$$\text{Maximize } Z = x_1 + 9x_2 + x_3$$

$$\text{subject to } x_1 + 2x_2 + 3x_3 \leq 9$$

$$3x_1 + 2x_2 + 2x_3 \leq 15$$

$$x_1, x_2, x_3 \geq 0$$

(d) Solve the dual of the following LPP :

$$\text{Minimize } Z = 10x_1 + 40x_2$$

$$\text{subject to } x_1 + 2x_2 \geq 2$$

$$-x_1 - x_2 \geq 1$$

$$x_1, x_2 \geq 0$$

- (e) Obtain an initial basic feasible solution to the following transportation problem by North-West Corner rule :

	D_1	D_2	D_3	D_4	
O_1	95	105	80	15	120
O_2	115	180	40	30	70
O_3	115	185	95	70	50
	50	40	40	110	

- (f) Solve the following minimal assignment problem :

	I	II	III	IV
A	2	3	4	5
B	4	5	6	7
C	7	8	9	8
D	3	5	8	4

4. Define convex set. Show that the set of all convex combinations of a finite number of points is a convex set. 10

OR

Find all the basic feasible solutions of the system of equations :

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$

$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$

5. Solve the following LPP by two-phase method :
10

$$\text{Maximize } Z = 5x_1 - 4x_2 + 3x_3$$

$$\text{subject to } 2x_1 + x_2 - 6x_3 = 20$$

$$6x_1 + 5x_2 + 10x_3 \leq 76$$

$$8x_1 - 3x_2 + 6x_3 \leq 50$$

$$x_1, x_2, x_3 \geq 0$$

OR

- Use Big-M method to solve the following LPP :

$$\text{Maximize } Z = x_1 + 2x_2 + 3x_3 - x_4$$

$$\text{subject to } x_1 + 2x_2 + 3x_3 = 15$$

$$2x_1 + x_2 + 5x_3 = 20$$

$$x_1 + 2x_2 + x_3 + x_4 = 10$$

$$x_1, x_2, x_3 \geq 0$$

6. State and prove the Fundamental theorem
of Duality.
10

OR

Solve the following transportation problem :

	D_1	D_2	D_3	
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
	7	9	18	

7. Solve the following assignment problem :

	I	II	III	IV
A	20	28	19	13
B	15	30	16	28
C	40	21	20	17
D	21	28	26	12

10

OR

Use Linear Programming method to solve the following game :

	B		
A	1	-1	3
	3	5	-3
	6	2	-2

OPTION - B

Paper : MAT-HE-5056

(Spherical Trigonometry and Astronomy)

Full Marks : 80

1. Answer the following questions: $1 \times 10 = 10$
 - (i) Define great circle and small circle.
 - (ii) Define hour angle of a heavenly body.
 - (iii) What is the point on the celestial sphere whose latitude, longitude, right ascension and declination, all are zero ?
 - (iv) Name the *two* points in which the ecliptic cuts the equator on the celestial sphere.
 - (v) What do you mean by circumpolar star ?
 - (vi) State the third law of Kepler.
 - (vii) Where does the celestial equator cut the horizon ?
 - (viii) Define right ascension of a heavenly body.
 - (ix) What are the altitude and hour angle of the zenith ?
 - (x) State the cosine formula related to a spherical triangle.

2. Answer the following questions : $2 \times 5 = 10$
- (a) State Newton's law of gravitation.
 - (b) ABC is an equilateral spherical triangle, show that $\sec A = 1 + \sec a$.
 - (c) Give the usual three methods for locating the position of a star in space.
 - (d) Prove that the altitude of the celestial pole at any place is equal to the latitude of the place of the observer.
 - (e) Prove that the altitude of the celestial pole at any place is equal to the latitude of that place.
3. Answer **any four** questions of the following : $5 \times 4 = 20$
- (a) In a spherical triangle ABC , prove that $\cos a = \cos b \cos c + \sin b \sin c \cos A$.
 - (b) In a spherical triangle ABC , if $b + c = \pi$, then prove that $\sin 2B + \sin 2C = 0$.
 - (c) At a place in north latitude ϕ , two stars A and B of declinations δ and δ_1 respectively, rise at the same moment and A transits when B sets. Prove that $\tan \phi \tan \delta = 1 - 2 \tan^2 \phi \tan^2 \delta_1$

- (d) If ψ is the angle which a star makes at rising with the horizon, prove that $\cos \psi = \sin \phi \sec \delta$, where the symbols have their usual meanings.
- (e) Deduce Kepler's laws from the Newton's law of gravitation.
- (f) Prove that the altitude of a star is the greatest when it is on the meridian of the observer.

4. Answer **any four** questions of the following :

$$10 \times 4 = 40$$

- (a) In a spherical triangle ABC , prove that

$$\frac{\sin a}{\sin A} = \sqrt{\frac{1 - \cos a \cos b \cos c}{1 + \cos A \cos B \cos C}}$$

- (b) If ψ is the angle at the centre of the sun subtended by the line joining two planets at distance a and b from the sun at stationary points, show that

$$\cos \psi = \frac{\sqrt{ab}}{a + b - \sqrt{ab}}$$

- (c) If the inferior ecliptic limits are $\pm \varepsilon$ and if the satellite revolves n times as fast as the sun, and its node regresses θ every revolution the satellite makes round its primary, prove that there cannot be fewer consecutive solar eclipses at one node than the integer next less than
$$\frac{2(n-1)\varepsilon}{n\theta + 2\pi}.$$

- (d) State Kepler's laws of planetary motion. If V_1 and V_2 are the linear velocities of a planet at perihelion and aphelion respectively and e is the eccentricity of the planet's orbit, prove that
$$(1-e)V_1 = (1+e)V_2.$$

- (e) Assuming the planetary orbits to be circular and coplanar, prove that the sidereal period P and the synodic period S of an inferior planet are related to the earth's periodic time E by
$$\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$$
 Calculate the sidereal period (in mean solar days) of a planet whose sidereal period is same as its synodic period.

- (f) What is Cassin's hypothesis? Under this hypothesis, show that the amount of refraction R can be found from

$$\tan \phi = \frac{\sin R}{\mu - \cos R}, \text{ where } \mu \text{ is the}$$

refractive index of the atmosphere with respect to vacuum and ϕ is the angle of refraction at certain point on the upper surface of the atmosphere.

- (g) On account of refraction, the circular disc of the sun appears to be an ellipse. Prove it.

- (h) Prove that, if the fourth and higher powers of e are neglected,

$$E = M + \frac{e \sin M}{1 - e \cos M} - \frac{1}{2} \left(\frac{e \sin M}{1 - e \cos M} \right)^3 \text{ is}$$

a solution of Kepler's equation in the form.

OPTION - C

Paper : MAT-HE-5066

(Programming in C)

Full Marks : 60

1. Answer the following: 1×7=7
- (a) 'C' is a object-oriented programming language. (State True or False)
- (b) Write down what the following will return-
- ```
int a[30];
size of (a);
```
- (c) What is the meaning of +21 and -7 ?
- (d) What is the relational operator for 'not equal to' ?
- (e) In C language a comment starts with the symbol \_\_\_\_\_ and ends with the symbol \_\_\_\_\_.  
*(Fill in the blanks)*
- (f) What does '\n' mean ?
- (g) What happens if the condition in a while loop is initially false ?

2. Answer the following questions :  $2 \times 4 = 8$

(a) Give the output for

```
printf("\n%d%d%d \n", i, ++i, i++)
```

(Assume  $i = 3$ )

(b) Explain printf ( ) function.

(c) What are the differences between break and exit ( ) function ?

(d) What is local variable and global variable ?

3. Answer **any three** from the following :

$5 \times 3 = 15$

(a) What is 'for loop' ? Write down the form of 'for loop'. Write a C program to check whether a given number is prime or not using 'for loop'.  $1 + 1 + 3 = 5$

(b) Explain with examples all the assignment operators. 5

(c) Differentiate between 'if-else' and 'nested if-else' statement. Write C program to find biggest of three numbers using if-else and nested if-else statement. (Write two programs separately)  $1 + 2 + 2 = 5$



(d) What is an array variable ? How does it differ from an ordinary variable ? How do you initialize arrays in C ?

2+1+2=5

(e) What is recursive function ? What are the uses of this function ? Write a C program to find the factorial of a given positive number using recursion.

1+2+2=5

4. (a) Write a C program to sort a set of  $n$  numbers in ascending order and explain the algorithm used.

5+5=10

**OR**

(b) Explain the unconditional control statements of C in detail.

10

5. (a) Explain the various types of functions supported by C. Give examples for each of the C functions. What are the rules to call a function ? What are actual and formal arguments ?

2+2+4+2=10

**OR**

(b) Explain the structure of C program in detail.

10

6. (a) Write a C program to compute  $\cos(x)$   
upto 15 terms. 10

**OR**

(b) Write C programs to add and multiply  
two matrices of order  $(3 \times 3)$ .