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MATHEMATICS

(Major)

Paper : MAT0500104

(Multivariate Calculus)

Full Marks : 60

Time : 2½ hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following as directed : 1×8=8

(a) If

$$f(x, y) = \frac{1}{\sqrt{9 - x^2 - y^2}}$$

then find the domain of it.

(b) Define level curve of a function $f(x, y)$ at a constant c .

(c) Find f_x if $f(x, y) = x^2 \sin(3x + y^3)$.

(d) If $f(x, y) = \tan xy$, then df is

(i) $y \sec^2 xy dy + x \sec^2 xy dx$

(ii) $y \sec^2 x dy dx + x \sec^2 xy dy$

(iii) $\sec^2 xy dx + \sec^2 xy dy$

(iv) $\sec^2 x dy + \sec^2 y dx$

(Choose the correct answer)

(e) If $P_0(x_0, y_0)$ is a critical point of $f(x, y)$ and f has continuous 2nd-order partial derivatives in a disk centered at (x_0, y_0) and $D = f_{xx} f_{yy} - f_{xy}^2$, then a relative maximum occurs at P_0 , if

(i) $D(x_0, y_0) > 0$ and $f_{yy}(x_0, y_0) > 0$

(ii) $D(x_0, y_0) > 0$ and $f_{yy}(x_0, y_0) < 0$

(iii) $D(x_0, y_0) < 0$ and $f_{yy}(x_0, y_0) > 0$

(iv) $D(x_0, y_0) < 0$ and $f_{yy}(x_0, y_0) < 0$

(Choose the correct answer)

(f) Find the Jacobian of the transformation from Cartesian coordinate to polar coordinate system.

(g) State when a vector field C is said to be conservative.

(h) Find curl of the vector field

$$\vec{V}(x, y, z) = u(x, y, z)\hat{i} + v(x, y, z)\hat{j} + w(x, y, z)\hat{k}$$

2. Answer any six of the following questions :

2×6=12

(a) Show that

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^4 + y^2}$$

does not exist.

(b) Compute the slope of the tangent line to the graph of $f(x, y) = x \ln(x + y^2)$ at the point $P_0(e, 0, -e)$ in the direction parallel to the XZ -plane.

(c) Find the critical point of

$$f(x, y) = (x-2)^2 + (y-3)^4$$

and classify them.

(d) Find $\iint_R x \sin xy dA$ where

$$R = \{(x, y) : 0 \leq x \leq \pi, 0 \leq y \leq 1\}$$

(e) If $\vec{F} = \vec{\nabla} f$ is the gradient of the function

$$f(x, y, z) = -\frac{1}{x^2 + y^2 + z^2}$$

then by using fundamental theorem of line integral, find the work done by $\vec{F}$ along the smooth curve C joining $(1, 0, 0)$ to $(0, 0, 2)$ that not passes through the origin.

- (f) Evaluate $\int_4^7 \int_1^2 \int_0^3 x^2 y^2 z^2 dx dy dz$.
- (g) Find a parametrization of the cylinder $x^2 + (y-3)^2 = 9$, $0 \leq z \leq 5$.
- (h) Find $\text{div } \vec{F}$, given that $\vec{F} = \nabla f$ where $f(x, y, z) = x^2 y z^3$.
- (i) State Green's theorem.
- (j) Evaluate $\int_C (x+y) dx$ if C be the curve represented by $x = 2t$, $y = 3t^2$, $0 \leq t \leq 1$.

3. Answer any four of the following questions :

5×4=20

- (a) Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$, $x = \frac{r}{s}$, $y = r^2 \ln s$, $z = 2r$.

- (b) Show that

$$f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

is continuous at every point except at the origin.

- (c) Find the equation of the tangent plane and the normal line at $P_0(1, -1, 2)$ on the surface S is given by $x^2y + y^2z + z^2x = 5$.
- (d) Find all critical points on the graph $f(x, y) = 8x^3 - 24xy + y^3$ and use the second partial test to classify as a relative extrema or a saddle point.
- (e) Find the work done by the force field $\vec{F} = (x^2 + y^2)\hat{i} + (x + y)\hat{j}$ as an object moves counter clockwise along the circle $x^2 + y^2 = 1$ from $(1, 0)$ to $(-1, 0)$ and then back to $(1, 0)$ along the x -axis.
- (f) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where
- $$\vec{F} = (y^2 - z^2)\hat{i} + 2yz\hat{j} - x^2\hat{k}$$
- and C is curve defined parametrically by $x = t^2, y = 2t, z = t$ for $0 \leq t \leq 1$.
- (g) Find the mass of a lamina of density $\delta(x, y, z) = z$ in the shape of the hemisphere $z = (a^2 - x^2 - y^2)^{\frac{1}{2}}$.
- (h) Evaluate the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) dy dx$$

4. Answer any *two* of the following questions :

10×2=20

- (a) (i) Suppose the function f is differentiable at the point P_0 and that the gradient at P_0 satisfies $\vec{\nabla} f_0 \neq 0$. Then show that $\vec{\nabla} f$ is orthogonal to the level surface of f through P_0 . 5

- (ii) Let $f(x, y, z) = xyz$ and let $\vec{u}$ be a unit vector perpendicular to both $\vec{v} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{w} = \hat{i} + \hat{j} - \hat{k}$. Find the directional derivative of f at $P_0(1, -1, 2)$ in the direction of $\vec{u}$. 5

- (b) Show that the vector field $\vec{F} = 2x(y^2 + z^2)\hat{i} + 2x^2y\hat{j} + 3x^2z^2\hat{k}$ is conservative. Also find its scalar potentials. 4+6=10

- (c) Evaluate

$$\iiint_D (x^2 + y^2 + z^2) dx dy dz$$

where D denotes the region bounded by $x = 0, y = 0, z = 0$ and $x + y + z = a, a > 0$. 10

- (d) Use the method of Lagrange's multiplier to minimize

$$f(x, y) = 16 - x^2 - y^2$$

subject to $x + 2y = 6$. 10

- (e) (i) State divergence theorem. By using divergence theorem, evaluate $\iint_S \vec{F} \cdot \vec{N} dS$ where

$$\vec{F} = x^2 \hat{i} + xy \hat{j} + x^3 y^3 \hat{k}$$

and S is the surface of the tetrahedron bounded by the plane $x+y+z=1$ and the coordinate planes with outward unit normal vector $\vec{N}$. 1+5=6

- (ii) Find the area of the region D bounded above the line $y=x$ and below by the circle $x^2 + y^2 - 2y = 0$. 4

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