2015

MATHEMATICS

(Major)

Paper : 1.1

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer/choose the correct option : $1 \times 10 = 10$

   (a) Give an example of a relation on the set of real numbers $\mathbb{R}$ which is reflexive and transitive but not symmetric.

   (b) Find all partitions of the set $x = \{1, 2, 3\}$.

   (c) Let Q_+ be the set of all positive rational numbers and $*$ be a binary operation on Q_+ defined by $a * b = \frac{ab}{3}$, $\forall$ a, b $\in Q_+$. Find the identity element of Q_+ and determine the inverse of nonzero element $a \in Q_+$.

[Turn over

(d) Define an operation $*$ on the set of real numbers $\mathbb{R}$ as $a * b = a + 2b$, $\forall$ a, b $\in \mathbb{R}$. Then $(\mathbb{R}, *)$ is not a group because

(i) $\mathbb{R}$ is not closed w.r.t $*$

(ii) $\mathbb{R}$ is not associative w.r.t $*$

(iii) Identity element does not exist.

(iv) Inverse of each nonzero element does not exist.

(e) The value of i^i is

(i) $e^{-(4n+1)\frac{\pi}{2}}$ (ii) $e^{(4n-1)\frac{\pi}{2}}$

(iii) $e^{(4n+1)\frac{\pi}{2}}$ (iv) $e^{-(4n-1)\frac{\pi}{2}}$

(f) If z is a complex number, then $\sin^{-1}z$ is

(i) $-i \log\left(iz \pm \sqrt{1-z^2}\right)$

(ii) $i \log\left(iz \pm \sqrt{1+z^2}\right)$

(iii) $-i \log\left(z \pm i\sqrt{1-\right.$

(iv) $i \log\left(z \pm i\sqrt{1+z\right.$

(g) If A is a square mat
is

(i) $2\,|A|$

(iii) null matrix

(h) Inverse of which of
exists ? Given that 1,
of unity.

(i) $\begin{pmatrix} 1 & \omega \\ \omega & \omega^2 \end{pmatrix}$

(ii) $\begin{pmatrix} \omega^2 & 1 \\ 1 & \omega \end{pmatrix}$

(iii) $\begin{pmatrix} \omega & \omega^2 \\ \omega^2 & 1 \end{pmatrix}$

(iv) $\begin{pmatrix} \omega & \omega^2 \\ \omega & 1+\omega \end{pmatrix}$

(i) The rank of the matrix

$$\begin{pmatrix} 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{pmatrix} \text{ is}$$

(i) 0 (ii) 1

(iii) 2 (iv) 3

(j) If $2+\sqrt{3}$ is a root of the equation $x^4+2x^3-16x^2-22x+7=0$, then the other three roots are

(i) $2-\sqrt{3},\ 3+\sqrt{2},\ -3+\sqrt{2}$

(ii) $-2-\sqrt{3},\ -3+\sqrt{2},\ -3-\sqrt{2}$

(iii) $2-\sqrt{3},\ -3+\sqrt{2},\ -3-\sqrt{2}$

(iv) $-2+\sqrt{3},\ 3+\sqrt{2},\ -3-\sqrt{2}$

2. Give answers to the following questions : $2\times5=10$

(a) If $f : A \to B$ and $g : B \to C$ are bijective mappings, then prove that gof is also a bijective mapping.

(b) In a group G, pro
$\forall$ a, b $\in$ G.

(c) Show that every re
Hermitian.

(d) Prove that

$$\pi = 2\sqrt{3}\left(1-\frac{1}{3\cdot3}\right.$$

(e) If α, β, γ are the
$x^3+px^2+qx+r =$

$$\sum\frac{1}{\alpha^2\beta^2}.$$

3. Answer any *four* parts :

(a) Define an equivalence
set.

Show that the relatio
m" is an equivalence
integers.

(b) Let $f : A \to B$, g :
be three mappings. F

(i) ho (gof) = (hog

(ii) foi = f and jof =
and $j : B \to B$

(c) If G is a group in which $(ab)^i = a^i b^i$ for three consecutive integers i and any a, b in G, then prove that G is abelian. 5

(d) (i) Find the values of $(1+i)^{\frac{1}{3}}$

(ii) Prove that

$$\sin^2\theta\cos\theta = \theta^2 - \frac{5}{6}\theta^4 + \ldots\ldots$$

$$+(-1)^{n+1}\frac{3^{2n}-1}{4(2n)!}\theta^{2n} + \ldots\ldots\ldots \qquad 2+3=5$$

(e) (i) Apply Descarte's rule of signs to ascertain the minimum number of complex root of the equation $x^7-3x^3-x+1=0$.

(ii) If α, β, γ are the roots of the equation $x^3+px^2+qx+r=0$, find the equation whose roots are $\alpha\beta+\beta\gamma$, $\beta\gamma+\gamma\alpha$ and $\gamma\alpha+\alpha\beta$. 3+2=5

(f) Define rank of a matrix. Find the rank of the matrix 1+4=5

$$\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

4. Answer any *one* part :

(a) (i) Prove that if G i
is a subgroup of
0 (G).

(ii) If G is a finite gr
$a \in G$, $a^{0(G)} = e$
element of the g

(iii) Prove that every
cyclic group is c

(b) (i) Prove that centre
subgroup of the

(ii) For any integer a
that $a^p \equiv a$ (mod

(iii) Prove that a finite
is abelian.

5. Answer any *one* part :

(a) (i) If z is a comple

$$|z| \geq \frac{1}{\sqrt{2}}(|\text{Rl } z| + |\text{Im}$$

(ii) Express log (x + iy) in the form A + iB where A and B are reals. Also find log (x + iy).

(iii) If $\cos^{-1}(\alpha + i\beta) = \theta + i\phi$, prove that $\alpha^2 \operatorname{sec h}^2\phi + \beta^2 \operatorname{cosec h}^2\phi = 1$

3+4+3=10

(b) (i) Two complex numbers z_1 and z_2 are such that $|z_1 + z_2| = |z_1 - z_2|$. Show that amp z_1 and amp z_2 differ by $\frac{\pi}{2}$ or $\frac{3\pi}{2}$.

(ii) Expand tan x in ascending powers of x.

(iii) Deduce the Gregory series

$$\theta = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5} + \tan^5\theta - \cdots\cdots\infty,$$

where $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$.

3+3+4=10

6. Answer any *two* parts : 5×2=10

(a) Find the value of k and solve the equation $8x^3 - 12x^2 - kx + 3 = 0$ if the roots are in arithmetic progression. 5

(b) Solve the equation x

if it have three distinct

(c) Solve the equation

given that two roots

relation $2\alpha + \beta = 3$.

(d) Solve the equation x^3

method.

7. Answer any *two* parts :

(a) Define a symmetric

matrices.

Show that the matrix

skew-symmetric accor

or skew-symmetric.

(b) If A is a square matri

$|\text{adj}(\text{adj}A)| = |A|^{(n-1)^2}$

(c) Prove that the nec

condition for a ma

inverse is that $|A| \neq 0$

(d) For what values of λ the equations

$$x + y + z = 1$$

$$x + 2y + 4z = \lambda$$

$$x + 4y + 10z = \lambda^2$$

have a solution and solve them completely in each case. 5

Total No. of printed pages = 7

3 (Sem 1) MAT M2

2015

MATHEMATICS

(Major)

Paper : 1.2

(Calculus)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following : 1×10=10

(a) Write down the *n*th derivative of $\log_e(ax+b)$.

(b) If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

(c) Find $\frac{ds}{dx}$ for the curve $y^2 = 4ax$.

[Turn over

(d) Find the radius of curvature at any point (s, Ψ) on the curve $s = c \log \sec \Psi$.

(e) Write down the asymptotes of the curve $x^2 - y^2 = a^2$.

(f) If $f(x, y) = x \cos y + y \cos x$, find f_{xy}.

(g) Choose the correct answer : $\int \frac{dx}{a^2 - x^2}$ equals

(i) $\frac{1}{a}\tan^{-1}\frac{x}{a} + c, a \neq 0$

(ii) $\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c, |x| \neq |a|$

(iii) $\frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c, |x| \neq |a|$

(iv) $\sin^{-1}\frac{x}{a} + c, |x| < |a|$

(h) Write down the value

(i) Evaluate $\int_0^{\frac{\pi}{2}} \cos^4 x \, dx$

(j) Write down the intri curve

$y = a \log \sec\left(\frac{x}{a}\right),$

the origin.

2. Answer the following que

(a) If $y = \cos^3 x$, find y_n

(b) Find $\frac{ds}{d\theta}$ for the curv

(c) Prove that $\int_0^{\frac{\pi}{2}} \sin 2x$ lo

(d) Find the area of a hype by the x-axis and the $x = b$.

(e) Find the volume of solid generated by revolving about the x-axis, the area bounded by $y = \sin x$; $x = 0$, $x = \pi$.

3. Answer the following questions : 5×4=20

(a) If $u = \log(x^3+y^3+z^3-3xyz)$, show that

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{3}{(x+y+z)^2}$$

Or

If $F(v^2-x^2, v^2-y^2, v^2-z^2) = 0$, where v is a function x, y, z, show that

$$\frac{1}{x}\frac{\partial v}{\partial x}+\frac{1}{y}\frac{\partial v}{\partial y}+\frac{1}{z}\frac{\partial v}{\partial z}=\frac{1}{v}$$

(b) Trace the curve

$$y^2 = x^2\frac{a+x}{a-x}$$

Or

Show that the portion of the tangent at any point on the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$, intercepted between the axes is of constant lengths.

(c) Evaluate $\int\frac{\tan x}{\sqrt{a+b\tan}}$

Or

$\int\frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$, β

(d) Show that the area b

$\sqrt{x}+\sqrt{y}=\sqrt{a}$ and t

$\frac{1}{6}a^2$.

4. Answer either (a) or (b)

(a) (i) If $u = \sin ax +$

$u_n = a^n[1+(-1)^n$

(ii) If $y = a\cos(\log$

prove that

$x^2 y_{n+2}+(2n+1)x$

(b) (i) State and prove th Homogeneous func

(ii) If $u = \phi(H_n)$, where H_n is a homogeneous function of degree n in x, y, z, then show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=n\frac{F(u)}{F'(u)}$$

where $F(u) = H_n$. 4

5. Answer either (a) or (b) :

(a) (i) Find the asymptotes of the curve

$y^3 + x^2y + 2xy^2 - y + 1 = 0$. 5

(ii) If ρ_1 and ρ_2 be the radii of curvature at the ends of a focal chord of the parabola $y^2 = 4ax$, then show that

$$\rho_1^{-\frac{2}{3}} + \rho_2^{-\frac{2}{3}} = (2a)^{-\frac{2}{3}}$$ 5

(b) Define double points and double cusp. Search for double points on the curve $x^4 - 2y^3 - 3y^2 - 2x^2 + 1 = 0$. 2+8=10

6. (a) If $u_n = \int_0^1 x^n \tan^{-1} x\, dx$, then prove that

$$(n+1)u_n + (n-1)u_{n-2} = \frac{\pi}{2} - \frac{1}{n}$$ 5

(b) If $J_n = \int \sec^n x\, dx$,

$(n-1)J_n = \tan x \sec^{n}$

7. (a) Find the total lengt

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$$

(b) Find the volume of revolving the cardioi the initial line.