2012

PHYSICS

(Major)

Paper : 3.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 25)

1. Choose the correct option/Answer the following : 1×3=3

(a) What is the modulus of the determinant of a unitary matrix?

(i) 1

(ii) 0

(iii) −1

(iv) None of these

(b) What is a Hermitian matrix?

(c) What is a skew-symmetric matrix?

2. Define conjugate transpose of a matrix. Show that

$$(AB)^+ = B^+ A^+$$ 1+1=2

3. Answer any *two* questions out of *(a)*, *(b)* and *(c)* :

(a) *(i)* For three Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

prove that $\sigma_i \sigma_j = i\sigma_k$, where i, j, k are cyclic permutations of indices. 3

(ii) Show that modulus of each eigenvalue of a unitary matrix is unity. 2

(b) *(i)* Verify that

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

is an orthogonal matrix. 2

(ii) Show that

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}^{-1}$$ 3

(Continued)

(c) What is a fram reference frame a r another frame b v velocity $\vec{\omega}$. If the p acceleration of a p represented by $\vec{r}$, tively, show that th particle in frame b i

$$\vec{f}_b = \vec{f}_a + 2\vec{\omega}$$

4. Answer *either (a)* and *(*

Eithe

(a) State Cayley-Hamil the characteristic

$$A = \begin{bmatrix} 1 & 1 \\ 3 & 1 \\ 2 & 3 \end{bmatrix}$$

and verify Cayley-

(b) Find the mutually vectors of the mat

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Or

(c) Show that the trace of a product of two matrices is independent of the order of multiplication. Also show that eigenvalues of a Hermitian matrix are all real and its eigenvectors corresponding to two distinct eigenvalues are orthogonal. 2+3=5

(d) For the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

determine a matrix P such that $P^{-1}AP$ is a diagonal matrix. 5

(Continued)

GROUP—

(Electrosta

(Marks : 3

5. Choose the correct o following :

(a) Electric field vector *I*

(i) rotational

(ii) irrotational

(b) What do you unders octupole?

(c) What is meant by ele

(d) Define electrical susc

6. Answer the following ques

(a) The electric field c dipole at a point dist on its perpendicul $1{\cdot}5\times10^{-11}$ volt/m. moment.

(b) Write down Poisson's

(c) What is a polar m molecular polarizabilit

7. Write down the integral as well as differential form of Gauss' law. Use this law to show that the expression for field strength at a distance r due to an infinite line charge is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}$$

where λ is linear charge density and r is the distance of the external point from the line charge. 1+1+3=5

Or

Show that the interaction energy of two dipoles of moments $\vec{p}_1$ and $\vec{p}_2$ is given by

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{\vec{p}_1\cdot\vec{p}_2}{r^3} - \frac{3}{r^5}(\vec{p}_1\cdot\vec{r})(\vec{p}_2\cdot\vec{r})\right]$$

where $\vec{r}$ is the radius vector joining the centres of the two dipoles. Hence derive the torque acting on any dipole due to the field of another dipole. 4+1=5

8. Answer any *two* questions out of *(a)*, *(b)*, *(c)* and *(d)* :

(a) *(i)* Show that the electric field due to an electric dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{1+3\cos^2\theta}$$

where θ is the angle between $\vec{r}$ and $\vec{p}$. 5

(Continued)

(ii) Show that th electrostatic fi given by

$U =$

where the syn usual meaning

(b) *(i)* State and pro theorem rega Laplace's equa

(ii) Use Laplace's potential inside

(c) A point charge is infinite plane earth the method of e calculate—

(i) surface charge the plane;

(ii) the force betwee charge.

An electron is at a an infinite plane co the force experience and the work don infinite distance conductor.

(d) (i) A spherical cavity is cut in a dielectric medium. Show that

$$\vec{E}_{\text{eff}} = \vec{E} + \frac{\vec{P}}{3\varepsilon_0}$$

where the symbols have got their usual meanings. 3

(ii) Deduce Clausius-Mosotti relation. Point out its limitation. 6+1=7

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2012

PHYSICS

(Major)

Paper : 3.2

(Current Electricity and Magnetostatics)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) What is the dimension of capacitance in terms of the fundamental quantities (M, L, T, I)?

(b) Write down the differential form of Faraday's law of electromagnetic induction.

(c) What is the SI unit of thermoelectric power?

(d) Show that when a voltage of sinusoidal waveform is applied across a capacitor, the current passing through it leads the voltage by 90°.

(e) Can a charged particle at rest be accelerated by applying a magnetic field? Justify your answer.

(f) Write down the Maxwell's equation of electromagnetism which shows that magnetic monopole does not exist.

(g) If a sinusoidal voltage is applied to a series *L-C-R* circuit, under what condition the circuit becomes purely resistive?

2. Answer the following questions : 2×4=8

(a) A d.c. voltage V is suddenly applied to a series *R-C* circuit. Calculate the time it takes in charging the capacitor to 0·993 V. (Given $R = 5\ k\Omega$ and $C = 2\ \mu F$)

(b) Draw the circuit diagram of Kelvin's double bridge for low resistance measurement.

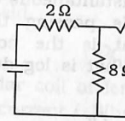
(c) An air-cored solenoid has a diameter of 2·5 cm and 500 turns wound over a length of 30 cm. Calculate the self-inductance of the solenoid. (Given permeability of air $\mu = 4\pi \times 10^{-7}$ H/m)

(d) Two parallel long straight wires at a distance 1 m apart placed in air carry equal currents $i = 5$ A in the same direction. Find the magnitude of force per unit length of the wires. (Given permeability of air $\mu = 4\pi \times 10^{-7}$ H/m)

(Continued)

3. Answer any *three* of the fo

(a) In the circuit give current and voltage resistor :



(b) A d.c. voltage of 80 V a circuit containing in series with an in Calculate the rate of g the instant when the and *(ii)* 16 A.

(c) The e.m.f. equation o couple (μV) is E where t is the temp between hot and cold in °C. Calculate the th and Peltier coefficient of the hot junction is junction is 0 °C.

(d) Starting from Ampere' $\oint \vec{B} \cdot d\vec{l} = \mu$ show that $\vec{\nabla} \times \vec{B} = \mu_0$ current density.

(e) Find the magnetic field at the centre of a circular coil carrying a steady current.

4. Establish the differential equation for a moving-coil ballistic galvanometer and find an expression for instantaneous deflection θ, when no charge is passing through the galvanometer. What is the condition for critical damping? What is log decrement in a ballistic throw? 7+2+1=10

Or

What are meant by 'self' and 'mutual' inductances? Find an expression for self-inductance of an air-cored long solenoid of radius a, length l and having N number of turns in it. 3+7=10

5. A series combination of an inductance L and a resistance R is connected in parallel with a lossless capacitor of capacitance C. A sinusoidal e.m.f. of amplitude V_0 and angular frequency ω is applied across the circuit. Find the resonant frequency and the impedance at resonance. Why is such a circuit known as rejector circuit? 6+2+2=10

Or

A d.c. e.m.f. E is suddenly applied to a circuit consisting of a resistance R, an inductance L and a capacitance C in series. Investigate the growth of charge in the circuit. Discuss the conditions for non-oscillatory and oscillatory growth of charge. 6+4=10

(Continued)

6. Differentiate between m
and magnetic scalar po
magnetic vector potenti
a very long thin stra
current I. Hence fin
magnetic field.

O

A rectangular coil of l
carrying a current i. W
in a uniform magnetic
the potential energy of
where $\vec{m}$ is the dipole m
carrying coil. Why can
coil be considered as a

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