2013

PHYSICS

(Major)

Paper : 3.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 25)

1. Answer the following : 1×3=3

(a) Define self-adjoint matrix.

(b) Define trace of a matrix.

(c) If A is a Hermitian matrix, show that e^{iA} is unitary.

2. Verify that $(AB)^T = B^T A^T$ where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix} \quad 2$$

3. Answer any *two* questions out of *(a)*, *(b)* and *(c)* :

(a) *(i)* If A and B are Hermitian matrices, show that $i(AB - BA)$ is also Hermitian. 1

(ii) If

$$A(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

then show that

$$A(\theta)\,A(\phi) = A(\phi)\,A(\theta) = A(\theta + \phi)$$ 2

(iii) Find the value of λ for which the matrix

$$A = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

will be orthogonal. 2

(b) *(i)* Show that inverse of the transpose of a square matrix is the transpose of the inverse. 2

(ii) Prove that any two eigenvectors corresponding to two distinct eigen-values of a unitary matrix are orthogonal. 3

(c) Show that for rot respect to anoth angular velocity motion of the coordinate syste

$$m\frac{d'^2\vec{r}}{dt^2} = \vec{F} - 2m($$

Which term repre term? Write the case of relative ro with non-uniform

4. Answer *either (a)* and

Eith

(a) *(i)* Prove that th unitary :

$$\begin{bmatrix} \frac{1}{2}(1+i) & \\ \frac{1}{2}(1+i) & \end{bmatrix}$$

(ii) Find the inv matrix from t

$$A = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

(b) (i) Show that every square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrix. 2

(ii) Solve by matrix method the following system of equations : 3

$$x+y+z=8$$
$$x-y+2z=6$$
$$3x+5y-7z=14$$

Or

(c) (i) Show that the eigenvalues of diagonal matrix are precisely the elements in the diagonal. 2

(ii) Given

$$A=\begin{bmatrix}1 & 1 & 2\\ 3 & 1 & 1\\ 2 & 3 & 1\end{bmatrix}$$

Compute A^{-1} by using the fact that A satisfies its characteristic equation. 3

(d) Find the eigenvalues and eigenvectors of the matrix

$$A=\begin{pmatrix}2 & -2 & 0\\ -2 & 1 & -2\\ 0 & -2 & 0\end{pmatrix}$$ 5

(Continued)

GROUP—

(Electrosta

(*Marks* :

5. Choose the correct o following :

(a) When a test charg infinity along the per of the dipole, the wo

(i) positive

(ii) zero

(iii) negative

(iv) None of the abo

(b) For a dipole, electric

(i) r^{-2}

(ii) r^{-3}

(iii) r^{-1}

(iv) r^{-4}

(c) The unit of $\vec{D}$ is

(i) V/m^2

(ii) $coul/m^2$

(iii) V/m

(iv) coul/m

(d) What is atomic polar

6. Answer the following questions : 2×3=6

(a) Can an electrostatic field have the form $\vec{E} = a(y\vec{a}_x - x\vec{a}_y)$, where a is a constant?

(b) Show that the function

$$\phi = 3x^2 + 8y - 3z^2$$

can represent the electrostatic potential in a charge-free region.

(c) Define relative permittivity. Write down Clausius-Mosotti relation.

7. Find an expression for the electric field at a point on the axis of a uniformly charged disc of radius a and surface charge density σ. Show that the electric field strength at point P inside a spherically symmetric charge distribution is given by

$$E_i = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3}$$

where R is the radius of the charge distribution and r is the distance of the internal point P from the centre of the charge distribution. 2+3=5

Find an expression f
one dipole on anothe
interaction potential
electric dipoles sepa
one of the dipoles is i
the radius vector join
the state of equilib
would make an angl

$$\tan\theta_2 =$$

8. Answer any *two* qu
(c) and *(d)* :

(a) *(i)* By using th
multipoles,
the electros
volume dist

(ii) An electron
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a total cha
Show that t
of the electr

$$U = \frac{}{4}$$

If this energy
m_0C^2 of the
its radius?

(b) (i) State and prove Poisson's equation in electrostatics. What form does it take when the charge density is zero? By solving Laplace's equation, show that the potential at a distance r from the axis of an infinitely long conducting cylinder of radius a_0 charged with a surface charge density σ is given by

$$\phi = -\frac{a_0\sigma}{\varepsilon_0}\ln\left(\frac{r}{a_0}\right)$$

Take the potential of the cylinder to be zero. 2+3=5

(ii) What is a polar molecule? Show that the electric intensity inside a parallel-plate capacitor whose dielectric constant increases linearly from the value k_1 in one plate to the value k_2 in other plate is given by

$$E(x) = \frac{\sigma/\varepsilon_0}{\left[\left(\frac{k_2 - k_1}{d}\right)x + k_1\right]}$$

where x is the distance from first plate, σ is the surface density of charge on first plate. The dielectric constant of a monatomic gas at NTP is 1·000538. If the gas is placed in an electric field of 30 kV/m, find the induced dipole moment of an atom. 1+2+2=5

(Continued)

(c) (i) What is mean
image? A poin
outside a gro
sphere. Find ar
intensity of the
the sphere by
electrical image

(ii) Define dielectri
dipole of length
placed at a dista
of an infinitely
charged wire ha
unit length. If th
plane containin
wire and the di
an angle θ with
the force on the
wire, then prove

$$\vec{F}_2 - \vec{F}_1 =$$

where distance

(d) (i) Deduce the di
Gauss' law for di
term electrical d

(ii) Show that for
placed in a hom
medium of inf
medium reduces
$+q$ to a value q
dielectric consta

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2013

PHYSICS

(Major)

Paper : 3.2

(Current Electricity and Magnetostatics)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Write down the continuity equation that relates the charge density and current density at a point.

(b) Consider two ballistic galvanometers with ballistic constants 2×10^{-5} coulomb/radian and 2×10^{-4} coulomb/radian. Which of these galvanometers is more sensitive? Justify your answer.

(c) If an electron initially moving in the x-direction is subjected to a magnetic field in the z-direction, in which direction the electron will be deflected?

(d) Express 'tesla' in terms of fundamental quantities (M, L, T, I).

(e) Why no power is dissipated if a voltage of sinusoidal waveform is applied across a purely inductive or capacitive circuit?

(f) Write down the differential form of Ampere's circuital law for steady currents.

(g) Distinguish between Joule's heating and Peltier's effect.

2. Answer the following questions : 2×4=8

(a) Establish the relation $V_{rms} = 0 \cdot 707\, V_0$, where V_{rms} and V_0 are the root mean square and peak values of voltage of sinusoidal waveform respectively.

(b) Draw the circuit diagram of Anderson's bridge for the measurement of inductance.

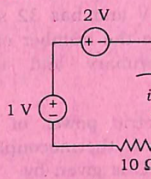
(c) The magnetic vector potential in some region is

$$\vec{A} = x^2 y\hat{i} + y^2 x\hat{j} - 4xyz\hat{k} \text{ Wb/m}$$

Find the magnetic field $\vec{B}$ at the point $(1m, 3m, -5m)$.

(Continued)

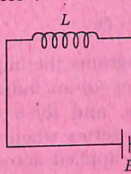
(d) From the circuit giv current i :



3. Answer any *three* of the fol

(a) A charge particle q revolving around a radius $r = 1$ Å with fre Calculate the magn magnetic moment at circular path. (Gi $\mu = 4\pi \times 10^{-7}$ H/m)

(b) The circuit shown bel where a steady curre to the time $t = 0$. Fin transient current flo circuit for $t > 0$:



(c) A single-phase 50 kVA transformer has primary voltage of 6600 V and secondary voltage of 256 V and has 32 secondary turns. Calculate the number of primary turns, and primary and secondary currents.

(d) The thermoelectric power of iron-lead and copper-lead thermocouples at a temperature t °C is given by

$$1734 - 4 \cdot 8t \ \mu V/°C \text{ and}$$

$$136 + 0 \cdot 95t \ \mu V/°C$$

respectively. Find the e.m.f. of a copper-iron thermocouple with junctions at 0 °C and 100 °C.

(e) Obtain an expression for the magnetic field near a straight wire of finite length carrying a steady current.

4. Set up the e.m.f. equation of series *R-L-C* circuit driven by a sinusoidal voltage. Solve the equation to find the instantaneous current. How does the impedance of the circuit behave at resonance? 2+6+2

Or

Find with vector diagrams the impedances of circuits consisting of *(a)* an inductance and resistance in series, and *(b)* a capacitance and a resistance in series when a source of alternating e.m.f. is applied across them. 5+5

(Continued)

5. A d.c. e.m.f. is suddenly consisting of a resistor *R* series. Write the e.m.f. equ and hence the current at meant by time constan significance?

Or

Explain the generation o fields in case of *(a)* two-p and *(b)* three-phase power principle of rotating magn to design induction moto

6. State and explain Bi magnetostatics. Using th obtain an expression for t a point on the axis of a carrying a steady current

Or

Define magnetic scalar p expression for the magne and hence magnetic fie carrying loop.

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