Acc No - 35 TO

3 (Sem–3) PHY M 1

2014

PHYSICS

(Major)

Paper : 3.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Physics)

(*Marks* : 25)

1. Answer the following questions : 1×3=3

(a) Define self-adjoint matrix.

(b) Show that trace of the sum of two matrices is the sum of their traces.

(c) Find the conjugate transpose of the following matrix :

$$A = \begin{pmatrix} 2+3i & -1+2i \\ i & 5-6i \end{pmatrix}$$

2. Show that the matrix A given by

$$A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$$

is unitary. 2

3. Answer any *two* of the following questions :

(a) *(i)* If A and B are two Hermitian matrices, then prove that AB is Hermitian only if A and B commute. 1

(ii) Solve the following system of equations by the use of matrix method : 2

$$x + 3y = 4$$
$$2x - 2y = 6$$

(iii) If

$$A = \begin{pmatrix} 1 & 2 & -2 \\ 4 & 1 & 5 \\ -3 & 2 & 4 \end{pmatrix}$$

then find B, when $A^T + 2B = 3I$. 2

(b) *(i)* Show that

$$A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

is a nilpotent matrix of index 2. 2

(Continued)

(ii) Prove th
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is unity.

(c) A reference fr
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b is given by

$$f_b = f_a$$

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b with non-ur

4. Answer *either* [*(a)*

(a) *(i)* Verify th

$A(a$

using

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(ii) For the Pauli spin matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \text{ and } \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

show that

$$[\sigma_1, \sigma_2] = 2i\sigma_3 \qquad 1\frac{1}{2}$$

(b) (i) Express the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & -2 \\ 4 & 2 & 0 \end{bmatrix}$$

as the sum of a symmetric and a skew-symmetric matrix. 2

(ii) Let the matrix

$$[A] = \begin{pmatrix} a & h \\ h & b \end{pmatrix}$$

is transformed to the diagonal form

$$[B] = T_\theta A T_\theta^{-1}$$

where

$$T_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Show that

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right) \qquad 3$$

(c) (i) What is a

(ii) By usin theorem,

(d) Diagonalize th

$A =$

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5. Choose the correc

(a) The energy d field is

(i) $\frac{E^2}{2\varepsilon}$

(ii) $\frac{\varepsilon E^2}{2}$

(iii) $\frac{2\varepsilon}{E^2}$

(b) The permittivity of a medium has the unit

(i) $\frac{F}{m}$

(ii) $F \cdot m$

(iii) $\frac{N}{m}$

(c) The electric field $\vec{E}$ and the electric potential ϕ are related by

(i) $\vec{E} = \vec{\nabla}\phi$

(ii) $\vec{E} = -\vec{\nabla}\phi$

(iii) $\phi = \vec{\nabla} \cdot \vec{E}$

(d) The dielectric constant K and the electrical susceptibility χ of a dielectric material are related by

(i) $K = 1 + \chi$

(ii) $\chi = 1 + K$

(iii) $K\chi = 1$

(Continued)

6. Answer the follo

(a) Check whe may be pos

$\vec{E} = (2x$

where A is dimensions. find how th with positio

Find $\vec{E}$ at (0 (0, 3, 0) m an

(b) If ρ' be th charges with slab placed prove that ρ

7. Using integral for statics, determin potential at a di infinitely long wire length.

Or

What is electric dipole? Show that the electric field in free space due to a dipole is given by

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0 r^3}\left[\frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^2} - \vec{p}\right]$$

where $\vec{p}$ is the dipole moment. 5

8. Answer any *two* questions :

(a) *(i)* Find an expression for the torque experienced by an electric dipole in external electric field. Hence show that the work done in rotating the dipole from an initial position θ_1 to the final position θ_2 is

$$W = -pE(\cos\theta_2 - \cos\theta_1)$$ 3+2=5

(ii) Find an expression for the potential energy due to the mutual interaction between two dipoles of dipole moments $\vec{p}_1$ and $\vec{p}_2$ respectively. Two water molecules each having a dipole moment $6{\cdot}2\times10^{-30}$ coulomb-metre point in the same direction and are inclined

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(b) *(i)* A unifor
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(ii) State a
theorem.

(iii) Show tha
spherical

$V = 4$

where *a* a
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(c) *(i)* Write Poi
Laplace's
potential a
axis of an
cylinder of
surface
potential o

(ii) Calculate with the method of electrical image the potential and the field at any point in space when a point charge is placed in front of a conducting plane of infinite extent maintained at zero potential. 5

(d) (i) Define electrical susceptibility. 1

(ii) An isotropic dielectric is placed in an otherwise uniform electrostatic field $\vec{E}$. Show that field inside a spherical cavity in this direction is

$$\vec{E}_i = \vec{E} + \frac{\vec{P}}{3\varepsilon_0}$$

where $\vec{P}$ is the polarization. 4

(iii) Establish the Clausius-Mosotti equation

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0}$$

for a linear dielectric material. 5

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2014

PHYSICS

(Major)

Paper : 3.2

(Current Electricity and Magnetostatics)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Write down the vector form of Ohm's law stating meaning of each term.

(b) Express 'Henry' in terms of fundamental quantities (M, L, T, I).

(c) Outline the principle used to build thermocouple thermometer.

(d) Write down the expression of magnetic flux density at the centre of a long solenoid carrying current.

(e) If a sinusoidal voltage is applied across a resistor, show that voltage and current are in phase.

(f) What does time constant signify in case of growth and decay of current in a d.c. circuit?

(g) Explain the term 'eddy current'.

2. Answer the following questions : 2×4=8

(a) Distinguish between 'magnetic vector potential' and 'magnetic scalar potential'.

(b) Explain the meanings of 'impedance' and 'reactance' of an a.c. circuit.

(c) Explain the differences between a 'dead-beat galvanometer' and 'ballistic galvanometer'.

(d) Draw the circuit diagram of Kelvin's double bridge for the measurement of low resistance.

3. Answer any *three* of the following questions : 5×3=15

(a) A current $i(t) = 2e^{-t} - e^{-2t}$ μA charges up a 120 nF capacitor for a period of 2s. If the final voltage across the capacitor is 15 V, what is the initial voltage across it?

(b) A coil of resis
0·1 H is co
capacitor with
a 200 V (r.m.
the power fac
the circuit.

(c) A coil of re
alternating c
(r.m.s.). The
identical coil
(r.m.s.) on o
inductance o
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(d) The charge o
value 1 μF fall
value in 5 min
the capacitor
resistance. W
resistance?

(e) Two exactly al
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sectional area
held at 20 °C
find the therm
rings.
[Given, for the
couple, $a = 4 \cdot 8$
resistivity
resistivity of co

4. What do you understand by 'self-' and 'mutual' inductances? Find an expression for the mutual inductance between the primary and secondary of a standard solenoid. Describe ballistic galvanometer and search coil method for the determination of mutual inductance. 2+4+4=10

Or

Explain Peltier and Thomson effects. Applying thermodynamics, derive the relations

$$\pi = T\frac{dE}{dT} \text{ and } \sigma_b - \sigma_a = T\frac{d^2E}{dT^2}$$

where the symbols have their usual meanings. 2+2+6=10

5. A d.c. e.m.f. is suddenly applied to a circuit consisting of a resistance R and a capacitor C in series. Investigate the growth of current in the circuit. Suppose after fully charging the capacitor, the d.c. e.m.f. is removed from the circuit. Now investigate the decay of current in the circuit. 5+5=10

Or

(a) Obtain an expression for the power factor of an a.c. circuit. Explain the term 'wattless current'. 3+2=5

(b) What is meant by resonance in an a.c. circuit? In an a.c. circuit containing L, C and R in series, find the condition under which the resonance is obtained. 2+3=5

(Continued)

6. A rectangular coil
carrying a current
a uniform magneti
potential energy
where $\vec{m}$ is the dip
carrying coil. Why
coil be considered

State and explain
Using Ampere's
expression for the
straight current-ca