2016

PHYSICS

(Major)

Paper : 3.1

(Mathematical Methods–III and Electrostatics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Physics)

(*Marks* : 25)

1. Answer the following questions : 1×3=3

(a) In matrices, find the value of $(A+B+C)^2$.

(b) Show that $(A^2)^{-1} = (A^{-1})^2$.

(c) What is the rank of a zero matrix?

2. Check whether

$$\begin{pmatrix} i/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & i/2 \end{pmatrix}$$

is a unitary matrix. 2

3. Answer any *two* of the following questions : 5×2=10

(a) *(i)* For an orthogonal matrix, if λ is an eigenvalue, what is the other value? 1

(ii) If

$$A_\alpha = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}, \quad A_\beta = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}$$

check whether $A_\alpha A_\beta = A_{\alpha+\beta}$ is correct or not. 2

(iii) If

$$A = \begin{array}{c} \begin{matrix} \textit{Room I} & \textit{Room II} \end{matrix} \\ \begin{pmatrix} 10 & 12 \\ 9 & 14 \\ 15 & 14 \end{pmatrix} \end{array} \begin{array}{l} \\ \textit{Flat I} \\ \textit{Flat II} \\ \textit{Flat III} \end{array}$$

gives the power consumed in two rooms within three flats and

$$X = \begin{pmatrix} 10 \\ 5 \end{pmatrix} \begin{array}{l} \textit{Room I} \\ \textit{Room II} \end{array}$$

gives the number of electrical items in rooms, then what information does $Y = AX$ yield and where is its highest value? 2

(Continued)

(b) *(i)* If

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$, then

What does t geometrically?

(ii) If A and B are H show that A Hermitian whe Hermitian.

(iii) Compute the ac

$$A = \begin{pmatrix} 0 \\ -1 \\ 5 \end{pmatrix}$$

(c) *(i)* Derive the expr F' acting on a rotating frame force F and t forces. Name th

(ii) What is the rotation of th acceleration du at a place whe

4. Answer *either (a), (b) or (c), (d)* : 5×2=10

(a) *(i)* If $A = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$

then find the value of A^n. 2

(ii) In an electrical network

$$I_1 - I_2 + I_3 = 0$$
$$2I_2 - 3I_3 = 0$$
$$5I_1 + 3I_2 = 2$$

Find the currents by matrix method. 3

(b) *(i)* If

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{pmatrix}, \quad A^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & c \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

then find the value of $a + c$. 3

(ii) If

$$A = \begin{pmatrix} 2 & -3 \\ 0 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$$

find $A - B$ and also a symmetric matrix out of it. 2

(c) *(i)* Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 3 & 0 & 1 \\ 2 & 3 & 1 \end{pmatrix}$$ 3

(Continued)

(ii) If $A = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

then using the

$A^2 - 5A$

find the value

(d) *(i)* Given

$x_1 = 3y_1$

$x_2 = -y_1$

Find the tran

for y_1, y_2 by r

(ii) If

$A = \begin{pmatrix} a \\ c \end{pmatrix}$

satisfies the e

$x^2 - (a+$

then find the

k, a,

GROUP—B

(Electrostatics)

(*Marks* : 35)

5. Choose the correct option : 1×3=3

(a) The relation $D = \varepsilon E$ is true for

(i) any medium

(ii) homogeneous medium

(iii) isotropic medium

(iv) homogeneous and isotropic medium

(b) Uniqueness of electric field strength E means

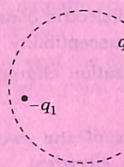
(i) $V_1 = V_2$

(ii) $\nabla V_1 = \nabla V_2$

(iii) $V_1 = V_2$ + constant

(iv) Both *(ii)* and *(iii)*

(c) A Gaussian surface
is shown by the dot



The electric field on t

(i) due to q_1, q_2 o

(ii) due to q_2 only

(iii) due to all

(iv) zero

6. Answer the following qu

(a) Given an electric fiel
surrounding an orig
for which the potent

$$\phi = ax^2$$

where a, C are pos
the components of
Where is the potenti
is the field intensity

Or

Show that $K = 1 + \chi$

where K = dielectric constant

χ = susceptibility

Define polarization charges. 2+1=3

(b) Check which of the two expressions below for electrical potential is applicable for a charged region. Correspondingly find the charge density : 2+1=3

(i) $3x^2 + y^2 + 2z^2$

(ii) $x^2 - y^2 + 8z$

7. Answer *either (a) or (b)* : 6

(a) *(i)* Find the electric field at a point located at a distance r_1 from the axis of a dipole of length d. Show that if $d/r_1 << 1$, the field at that behaves as $E = 2p/r_1^3$, p = dipole moment. 3

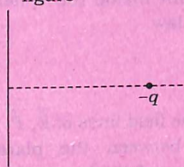
(ii) Define equipotential surface. What is the direction of electric field at a point on equipotential surface? 3

(b) *(i)* A sphere of radiu
charged by ch
Calculate the elec
the sphere.

(ii) Show that the div
field of a point ch

8. Answer any *two* questions

(a) *(i)* An electric di
2 mm having
$q = 2 \cdot 0 \times 10^{-8}$ C
a long line ch
$4 \cdot 0 \times 10^{-4}$ cm^{-1}
figure



such that the neg
a distance of 20
charge, the forc
dipole is 0·11k N

(ii) Establish the bou
satisfied by elec
electric displacem
the boundary
dielectrics.

(b) (i) Using Laplace's equation, show that the electric field is constant in the region between the two parallel plates and it is toward the plate of lower potential. 5

(ii) There is a solid sphere of radius R having volume charge density

$$\rho = \rho_0 (1 - r/R)$$

where ρ_0 is constant and r is the distance from the centre of the sphere. Find the electric intensity E at a point inside the sphere using Gauss' law. 5

(c) (i) Draw the field lines of $\vec{E}$, $\vec{P}$, $\vec{D}$ in the region between the plates of a capacitor (of thickness d) with the dielectric (of thickness t) in between the plates (given $d > t$). Show that $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$. 3+2=5

(ii) Deduce the relation between dielectric constant of a fluid and its polarizability. 5

(d) (i) Define electrical value of surface induced charge conducting plane charge. Draw the State the region equation is satisfi

(ii) An electron is dis infinite plane co the force experien

★ ★ ★