2019

PHYSICS

(Major)

Paper : 4.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods—IV)

(*Marks* : 40)

1. Answer the following questions : 1×4=4

(a) Write a second-order, second-degree differential equation.

(b) What is the value of $P_n(t)$?

(c) If $H_n(x)$ is the Hermite polynomial, then what is the value of $H_0(x)$?

(d) If one card is drawn at random from a pack of 52, what is the probability of getting a king?

2. Answer the following questions : 2×3=6

(a) Check whether Frobenius method can be applied or not to the following equation :

$$2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0$$

(b) Using the generating function for the Legendre polynomials

$$G(x, t) = (1 - 2xt + t^2)^{-1/2} = \sum_0^\infty P_n(x) t^n, \quad -1 \le x \le 1, \ t < 1$$

show that

$$P_n(-x) = (-1)^n P_n(x)$$

(c) What is the chance that a leap year selected at random will contain 53 Sundays?

Or

A function $f(x)$ is defined as

$$\begin{cases} 0 & , \quad x < 2 \\ \frac{1}{18}(2x+3), & 2 \le x \le 4 \\ 0 & , \quad x > 0 \end{cases}$$

Show that it is a probability density function.

3. Answer the following questions : 5×2=10

(a) Prove that

$$\int_{-1}^{+1} x P_n(x) P_{n-1}(x)\, dx = \frac{2n}{4n^2 - 1}$$

(b) Prove that

$$\int_{-\infty}^{+\infty} e^{-x^2} H_n^2(x)\, dx = 2^n \sqrt{\pi}\, .n!$$

Or

Calculate the mean and standard deviation for the following :

Size of item	:	6	7	8	9	10	11	12
Frequency	:	3	6	9	13	8	5	4

4. Answer the following questions : 10×2=20

(a) *(i)* If the solution $y(x)$ of Hermite's differential equation is written as

$$y(x) = \sum_{r=0}^{\infty} a_r x^{k+r}$$

then show that the allowed values of k are zero and one only. 5

(ii) Prove the recurrence relation

$$2nH_{n-1}(x) = H_n'(x)$$ 5

Or

(b) *(i)* For the Legendre polynomials, show that

$$\int_{-1}^{+1} P_m(x) P_n(x) dx = \frac{2}{2m+1} \delta_{m,n}$$

where

$$\delta = \begin{cases} 0, & \text{for } m \neq n \\ 1, & \text{for } m = n \end{cases}$$

7

(ii) Express

$$f(x) = x^4 - 2x^3 + 3x^2 + 5x - 9$$

in terms of Legendre polynomials. 3

(c) *(i)* Find power series solution of

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

5

(ii) Expand the function

$$f(x) = \begin{cases} -1, & \text{for } x < 0 \\ 0, & \text{for } x = 0 \\ 1, & \text{for } x > 0 \end{cases}$$

in terms of Hermite polynomials. 5

Or

(d) (i) Find the probability distribution of number of doublets in three throws of a pair of dice. 5

(ii) If two variates x and y are distributed normally, prove that their sum $x+y$ is also distributed normally. 5

GROUP—B

(Introduction to Computer and Computer Programming)

(*Marks* : 20)

5. Answer the following questions : 1×3=3

(a) Define the term 'computer program'.

(b) Define a flowchart.

(c) What is data type?

6. What are the different types of logical operations in C/C++/FORTRAN? 2

7. Answer any *one* of the following questions : 5

(a) An examination was conducted in a school. The gradings are declared as follows :

If

marks ≥ 75, declare as D

≥ 60, but < 75, declare as A

≥ 50, but < 60, declare as B

≥ 40, but < 50, declare as C

and < 40, declare as F

Draw a flowchart to perform this job, assuming that the number of students who appeared for the examination is 120.

(b) Write a program in C/C++/FORTRAN to find whether a given number is odd or even.

8. Answer any *one* of the following questions : 10

(a) *(i)* Write the algorithm for finding the solution of the following pair of equations in x and y :

$$a_1 x + a_2 y = b_1$$
$$a_3 x + a_4 y = b_2$$

Assume that a_1, a_2, a_3, a_4, b_1 and b_2 are non-zero. 5

(ii) Draw the flowchart for outputting the Fibonacci series 0, 1, 1, 2, 3, 5, 8, etc. for 20 terms. 5

(b) (i) Write a program in C/C++/ FORTRAN to convert a given temperature in Fahrenheit to Celsius using the following conversion formula : 5

$$C = \frac{F - 32}{1 \cdot 8}$$

(ii) Write a program in C/C++/ FORTRAN to find the largest of three distinct numbers X, Y and Z. 5

★★★

2019

PHYSICS

(Major)

Paper : 4.2

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Wave Optics)

(*Marks* : 40)

1. Answer the following questions : 1×4=4

(a) Why are Newton's rings circular in shape?

(b) Define specific rotation.

(c) Write the formulae for the resolving power of plane transmission grating.

(d) Yellow light is used in a single slit diffraction experiment with slit width of 0·6 mm. If yellow light is replaced by X-rays, how will the diffraction pattern be affected?

2. (a) Two waves from two incoherent sources are superposed. Show that the resultant intensity is the sum of the individual intensities. 2

(b) Monochromatic light of wavelength 5000 Å is diffracted by a grating of 2500 lines per cm. Show whether 16th order diffraction is possible. 2

(c) Calculate the thickness of half-wave plate for sodium light ($\lambda = 5893$ Å), given $\mu_0 = 1{\cdot}54$ and the ratio of velocity of ordinary and extraordinary components is 1·007. 2

3. Answer any *two* of the following questions : 5×2=10

(a) In a biprism arrangement, show that the distance between the virtual images of the source is $d = 2x(\mu - 1)\alpha$, where x is the distance of the source from the biprism base, α the refracting angle of the prism and μ is the refractive index of the material of the prism.

(b) A zone plate is made by arranging the radii of the circles which define the zones such that they are the same as the radii of Newton's rings formed between a plane surface and the surface having radius of curvature 2 m. Calculate the principal focal length of the zone plate.

(c) Two spectral lines with average wavelength 6000 Å are resolved in second order by a grating having 500 lines per cm. The least width of the grating is 2 cm. Find the difference in wavelength of the lines.

4. *(a)* *(i)* Draw a neat ray diagram for the experimental arrangement of Newton's rings arrangement. Deduce the relation

$$\lambda = \frac{D_{m+p}^2 - D_m^2}{4pR}$$

for Newton's rings, where D_m and D_{m+p} are the diameters of the mth and $(m+p)$th bright rings, R is the radius of curvature. 1+4=5

(ii) Narrate the construction and working of Laurent half-shade polarimeter. 5

Or

(b) *(i)* Explain the formation of colours in thin films. Show that the condition of bright and dark fringe for reflected light is just opposite to the condition for transmitted light. 6

(ii) A single slit is illuminated by light composed of two wavelengths λ_1 and λ_2. One observer observes that due to Fraunhofer diffraction, the first minima obtained for λ_1 coincides with the second diffraction minima of λ_2. Show that $\lambda_1 = 2\lambda_2$. 4

5. *(a)* Explain the Fresnel's diffraction due to a straight edge. Show that the separation between successive maxima goes on decreasing as we move away from the region of geometrical shadow. 5+1=6

(b) Distinguish among plane, circular and elliptical polarizations. How would you produce and detect circularly polarized light? 2+2=4

Or

(a) Discuss how the Michelson interferometer can be used to measure the wavelength of light. 6

(b) Find the value of half-angular width of the principal maxima for a single slit. 4

GROUP—B

(Special Theory of Relativity)

(*Marks* : 20)

6. Answer the following questions : 1×3=3

(a) What is Lorentz-FitzGerald contraction?

(b) A body moves with a velocity 0·2 c. Find the ratio of the moving mass to rest mass of the body.

(c) State the postulates of special theory of relativity.

7. (a) Write mass energy equivalence. Also give its physical significance. 2

(b) What is rest mass of a particle? Show that the rest mass of a particle is given by

$$m_0 = \frac{p^2c^2 - k^2}{2kc^2}$$

where p and k denote the momentum and kinetic energy of the particle. 1+4=5

8. (a) (i) Write the inverse Lorentz transformation equation for space and time. Using it derive the formula for time dilation. 2+3=5

(ii) Write brief notes on space-like and time-like intervals. 5

Or

(b) *(i)* Describe the Michelson-Morley experiment and explain the physical significance of negative results. 5+2=7

(ii) A constant force F is acted upon a particle of mass m and moving with velocity v. Show that the acceleration of the particle is

$$a = \frac{F}{m}\left(1 - \frac{v^2}{c^2}\right)^{3/2}$$ 3

★★★