2013

PHYSICS

(Major)

Paper : 5.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 30)

1. For $z = \frac{1+i}{(2-3i)^2}$

   (a) find Re z and Im z

   (b) find Mod z

   (c) find arg z

   (d) give the graphical representation of z. $1\times4=4$

2. (a) Find the roots of $\sqrt[3]{i}$ and locate them graphically. 2

(b) Define equivalent contour. 2

Or

Find the value of $(1+i)^5$.

3. (a) Determine if the following functions are analytic : 4

(i) $\dfrac{1+z}{1-z}$

(ii) e^{iz}

(b) Using Cauchy's integral formula, find the value of the integral

$$I = \oint \frac{z^2}{z^2-1}\,dz$$

around the unit circle at (i) $z=1$, (ii) $z=-1$. 4

Or

Find the Taylor series expansion about the origin for $f(z)=\dfrac{1}{(1-z)^m}$ and hence find the series for $\phi(z)=\dfrac{1}{1-z}$.

4. (a) Evaluate the integral

$$I = \int_0^{2\pi} \frac{d\theta}{5 - 4\sin\theta}$$ 7

(b) For a function $f(z)$ which has a pole of order m at $z = z_0$, show that the residue of the function at that singular point is

$$a_{-1} = \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} [(z - z_0)^m f(z)]_{z=z_0}$$

hence find the singular points and calculate the residues for $f(z) = \dfrac{e^z}{(z-2)^3}$.

5+2=7

Or

State and derive the Cauchy-Riemann conditions and then use them to compute the first derivative of $f(z) = e^z$. 5+2=7

GROUP—B

(Classical Mechanics)

(*Marks* : 30)

5. Answer the following questions : 1×4=4

(a) State Hamilton's principle.

(b) State one advantage of Lagrangian formulation over Newtonian formulation.

(c) A system of ten particles has five holonomic constants. How many generalised coordinates are required to describe the motion?

(d) What is virtual work? State the principle of virtual work.

6. (a) A Lagrangian is given by

$$L = \frac{1}{2}\alpha\dot{q}^2 - \frac{1}{2}\beta q^2$$

where α and β are constants. Find the Hamiltonian of the system. 2

(b) A particle moves in a circular orbit obeying inverse square law. Show that its angular momentum varies as the square root of its radius. 2

Or

What is a cyclic coordinate? Show that a cyclic coordinate in Lagrangian is also a cyclic coordinate in Hamiltonian.

7. Answer any *two* of the following questions : 4×2=8

(a) Establish d'Alembert's principle.

(b) Set up the Lagrangian of a compound pendulum and obtain its equation of motion.

(c) Deduce an expression of reduced mass of a two-body central force problem.

8\. (a) (i) Set up the differential equation of the orbit of a particle under the influence of a central force $F(r)$.

(ii) Show that if the position vector of a particle is given by $r = a\sin\theta$, then $F(r) \propto \frac{1}{r^5}$. 4+3=7

(b) If the Lagrangian of a conservative system does not contain time explicitly, show that the total energy of the system is conserved. Using Lagrange's equation, show that $F_x = -\frac{\partial V}{\partial x}$. 5+2=7

Or

Define Hamiltonian of a system and then derive Hamilton's canonical equations. 2+5=7

★★★