2018

MATHEMATICS

(Major)

Paper : 5.5

(Probability)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×8=8

(a) Write the condition for the outcomes of a random experiment so that $p+q=1$; p and q being the probability of success and failure respectively.

(b) If S is the sample space in a random toss of 7 coins, then write the number of elements of S.

(c) Is the probability mass function

x	-1	0	1
$p(x)$	0·4	0·4	0·3

admissible? Give reason.

(d) Sketch the area under any probability curve with probability density function $p(x)$ between $x = c$ and $x = d$ represented by $P(c \le X \le d) = \int_c^d p(x)\, dx$.

(e) For a discrete random variable X with probability function $p(x)$, rth moment about A is $\sum (x - A)^r p(x)$. What are the values of r and A for *(i)* $E(X)$ and *(ii)* $\text{var}(X)$?

(f) The density function of a random variable X is given by

$$f(x) = \begin{cases} 2, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Find $E(X)$.

(g) What is the probability of getting exactly 3 heads in 6 tosses of a fair coin?

(h) Name the distribution in which mean is the square of its standard deviation.

2. Answer any *four* of the following : 3×4=12

(a) If A and B are two possible outcomes of an experiment and $p(A) = 0 \cdot 4$, $P(A \cup B) = 0 \cdot 7$ and $P(B) = p$, then for what value of p, A and B become independent?

(b) A random variable X has the following probability function :

x	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Evaluate $P(X \geq 6)$ and $P(0 < x < 5)$.

(c) Show that in a frequency distribution

$$(x_i, f_i); i = 1, 2, \cdots, n$$

mathematical expectation of the random variable is nothing but its arithmetic mean.

(d) Define Poisson distribution and hence prove that $\sum_{r=0}^{\infty} p(r) = 1$.

(e) If the random variable X is normally distributed with mean μ and variance σ^2, show that the mean of the variate

$$z = \frac{x - \mu}{\sigma}$$

is always zero.

3. Answer any *two* from the following : 5×2=10

(a) Prove that two events A and B are independent $\Leftrightarrow P(A \cap B) = P(A)\, P(B)$.

(b) A man has five coins, one of which has two heads. He randomly takes out a coin and tosses in three times. What is the probability that it will fall head upward all the times?

(c) For two independent events A and B, prove that *(i)* A and $\overline{B}$ are independent and *(ii)* $\overline{A}$ and $\overline{B}$ are independent.

4. Answer any *two* from the following : 5×2=10

(a) Let X and Y be two random variables each taking three values −1, 0, 1 and having the joint probability distribution as given in the following table :

Y \ X	−1	0	1
−1	0	·1	·1
0	·2	·2	·2
1	0	·1	·1

Obtain the marginal probability distribution of X and Y.

(b) The probability function of a random variable X is given by

$$f(x) = \begin{cases} x^2/81, & -3 < x < 6 \\ 0, & \text{otherwise} \end{cases}$$

Find the probability density function for the random variable

$$u = \frac{1}{3}(12 - X)$$

(c) A random variable X has density function

$$f(x) = \begin{cases} ce^{-3x}, & x > 0 \\ 0, & x \le 0 \end{cases}$$

Find *(i)* the constant c, *(ii)* $P(1 < x < 2)$ and *(iii)* $P(X \ge 3)$.

5. Answer any *two* from the following : 5×2=10

(a) Prove that

$$\text{var}(ax + by) = a^2 \text{ var}(x) + b^2 \text{ var}(y) + 2ab \text{ cov}(x, y)$$

(b) A random variable has the following probability distribution :

x	0	1	2	3
$p(x)$	0·1	0·3	0·4	0·2

Find *(i)* $E(X)$ and *(ii)* var (X).

(c) A continuous random variable X has the probability function given by

$$f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & x \le 0 \end{cases}$$

Find *(i)* $E(X)$ and *(ii)* $E(X^2)$.

6. Answer any *two* from the following : 5×2=10

(a) Prove that for the binomial distribution with parameter n and p, variance cannot exceed $\frac{n}{4}$.

(b) Derive Poisson distribution as a limiting case of binomial distribution.

(c) Prove that the mean and variance of a binomially distributed variable are respectively $\mu = np$ and $\sigma^2 = npq$.

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