2018

MATHEMATICS

(Major)

Paper : 5.6

(Optimization Theory)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions as directed : $1 \times 7 = 7$

(a) Given a system of m simultaneous linear equations in n unknowns $(m < n)$, the number of basic variables will be

(i) m

(ii) n

(iii) $n - m$

(iv) $n + m$

(Choose the correct option)

(b) Express the vector $x = (5, 9)$ as the linear combination of the vectors $\alpha = (1, 2)$, $\beta = (3, 4)$.

(c) Define a line segment joining the points x and y in $\mathbb{R}^2$.

(d) The set of all feasible solutions of an LPP is a _____ set.

(Fill in the blank)

(e) In standard form of an LPP, all the constraints are expressed in the form of equations, except for the non-negative restrictions.

(State True or False)

(f) A necessary and sufficient condition for BFS to a maximization LPP to be an optimum is (for all j)

(i) $z_j - c_j \geq 0$

(ii) $z_j - c_j \leq 0$

(iii) $z_j - c_j = 0$

(iv) $z_j - c_j > 0$ or < 0

(Choose the correct option)

(g) Which of the following is not a convex set?

(i) $\{(x_1, x_2) : x_1^2 + x_2^2 = 1\}$

(ii) $\{(x_1, x_2) : |x_1| \leq 1, |x_2| \leq 1\}$

(iii) $\{(x_1, x_2) : x_1^2 + (x_2 - 1)^2 \leq 4\}$

(iv) None of the above

(Choose the correct option)

2. Answer the following questions : $2 \times 4 = 8$

(a) Show that a hyperplane in $\mathbb{R}^n$ is a convex set.

(b) Define the convex hull of a set $A \subseteq \mathbb{R}^n$. Determine the convex hull of the set $A = \{x_1, x_2\}$.

(c) Prove that $x_1 = 2$, $x_2 = -1$ and $x_3 = 0$ is a solution but not a basic solution to the system of equations

$$3x_1 - 2x_2 + x_3 = 8$$
$$9x_1 - 6x_2 + 4x_3 = 24$$

(d) Write the dual of the following primal problem :

$$\text{Minimize } Z = 5x_1 + 3x_2$$

subject to

$$3x_1 + 5x_2 = 12$$
$$5x_1 + 2x_2 = 10$$

with $x_1 \geq 0,\ x_2 \geq 0$

3. Answer any *three* parts of the following :

5×3=15

(a) Three different types of trucks A, B and C have been used to transport a minimum of 60 tons solid and 35 tons liquid substance. A type truck can carry 7 tons solid and 3 tons liquid. B type truck can carry 6 tons solid and 2 tons liquid and C type truck can carry 3 tons solid and 4 tons liquid. The costs of transport are ₹ 500, ₹ 400 and ₹ 450 per truck of A, B and C type respectively. Formulate the problem mathematically so that the total transportation cost is minimum.

(b) What is a balanced transportation problem? Describe a transportation table. Write the names of three common methods to obtain an initial basic feasible solution for a transportation problem.

1+1+3=5

(c) Solve graphically the following linear programming problem :

$$\text{Maximize } Z = 5x_1 + 7x_2$$

subject to

$$3x_1 + 8x_2 \leq 12$$
$$x_1 + x_2 \leq 2$$
$$2x_1 \leq 3$$

with $x_1 \geq 0,\ x_2 \geq 0$

(d) Prove that the set of all convex combinations of a finite number of points of $S \subseteq \mathbb{R}^n$ is a convex set.

(e) Find out all the basic solutions of the equations :

$$2x_1 + 3x_2 + x_3 = 8$$
$$x_1 + 2x_2 + 2x_3 = 5$$

and prove that one set of solution is not feasible.

4. Solve the following LPP by simplex method : 10

$$\text{Maximize} \quad Z = 3x_1 + 2x_2 + 5x_3$$

subject to

$$x_1 + 2x_2 + x_3 \leq 430$$
$$3x_1 + 2x_3 \leq 460$$
$$x_1 + 4x_2 \leq 420$$

with $x_1,\ x_2,\ x_3 \geq 0$

Or

Solve the following by two-phase method : 10

Maximize $Z = 5x_1 + 3x_2$

subject to

$$3x_1 + x_2 \leq 1$$
$$3x_1 + 4x_2 \geq 12$$

with $x_1, x_2 \geq 0$

5. Use Charnes Big-M method to solve the following LPP : 10

Maximize $Z = 3x_1 - x_2$

subject to

$$2x_1 + x_2 \geq 2$$
$$x_1 + 3x_2 \leq 3$$
$$x_2 \leq 4$$

with $x_1, x_2 \geq 0$

Or

Use duality to solve the following : 10

Minimize $Z = 3x_1 + x_2$

subject to

$$2x_1 + 3x_2 \geq 2$$
$$x_1 + x_2 \geq 1$$

with $x_1, x_2 \geq 0$

6. Solve the following transportation problem by using Vogel's approximations method for determination of IBFS and show that the optimal solution is degenerate : 10

	D_1	D_2	D_3	D_4	a_i
O_1	10	20	5	7	15
O_2	18	9	12	8	25
O_3	15	14	16	18	5
b_j	5	15	15	10	

Or

A company has 4 machines to do 3 jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table :

		Machine			
		W	*X*	*Y*	*Z*
	A	18	24	28	32
Job	*B*	8	13	17	19
	C	10	15	19	22

Assign the jobs to different machines so as to minimize the total cost. 10

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