2017

MATHEMATICS

(Major)

Paper : 5.4

(Rigid Dynamics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) Write down the moment of inertia of a circular ring of mass M and radius a about a diameter.

   (b) Define equimomental systems.

   (c) State D'Alembert's principle.

   (d) What is the conservation law of angular momentum of a rigid body?

   (e) Define the centre of percussion.

   (f) What are generalized coordinates?

   (g) Write down the degree of freedom of a rigid body moving with one point fixed.

   1×7=7

2. *(a)* Prove the perpendicular axes theorem.

(b) A rigid body with one point fixed rotates with angular velocity $\vec{\omega}$ and has angular momentum $\vec{\Omega}$. Prove that the kinetic energy is given by

$$T = \frac{1}{2}(\vec{\omega} \cdot \vec{\Omega})$$

(c) A particle moves under the influence of a central force field

$$-\frac{\mu}{r^3}\vec{r}$$

with the point O as the centre of force. Find the potential energy of the particle.

(d) Find the moment of inertia of a solid circular cylinder of radius a, height h and mass M about the axis of the cylinder. 2×4=8

3. *(a)* Show that the moment of inertia of an ellipse of mass M and semi-axes a and b about a tangent is

$$\frac{5M}{4}p^2$$

where p is the perpendicular from the centre on the tangent. 5

Or

Find the moment of inertia of a truncated cone about its axis, the radii of its ends being a and b. 5

(b) Prove that the centre of inertia of a body moves as if the whole mass of the body were collected at it and as if all the external forces acting on the body were acting on it in directions parallel to those in which they act. 5

(c) Obtain the equation of motion for a compound pendulum. 5

Or

An elliptic area of eccentricity e is rotating with angular velocity ω about one latus rectum, suddenly this latus rectum is loosed and the other fixed. Show that the new angular velocity is

$$\omega\frac{1-4e^2}{1+4e^2}$$ 5

4. Answer *either* (a) and (b) *or* only (c) :

(a) A uniform rod OA of length $2a$, free to turn about its end O, revolves with uniform angular velocity ω about the vertical OZ through O and is inclined at a constant angle α to OZ. Show that the value of α is either zero or

$$\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$$ 6

(b) A circular board is placed on a smooth horizontal plane and a body runs round the edge of it at a uniform rate. What is the motion of the centre of the board? 4

Or

(c) A rod of length $2a$, is suspended by a string of length l, attached to one end, if the string and rod revolve about the vertical with uniform angular velocity and their inclinations to the vertical be θ and ϕ respectively, show that

$$\frac{3l}{a} = \frac{(4\tan\theta - 3\tan\phi)}{(\tan\phi - \tan\theta)}\frac{\sin\phi}{\sin\theta}$$ 10

5. A uniform rod is held in a vertical position with one end resting upon a perfectly rough table and when released rotates about the end in contact with the table. Find the motion. 10

Or

A uniform sphere rolls down an inclined plane rough enough to prevent any sliding; discuss the motion. 10

6. Derive Lagrange's equation for holonomic system. 10

Or

A homogeneous sphere of radius a rotating with angular velocity ω about a horizontal diameter is gently placed on a horizontal table whose coefficient of friction is μ. Show that there will be slipping at the point of contact for a time

$$\frac{2}{7}\frac{\omega a}{\mu g}$$

and that then the sphere will roll with angular velocity

$$\frac{2\omega}{7}$$ 10

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